



MATHS

MADE eJEE

Class Notes in Handwritten Format

Class 11

JEE Main & Advanced

Handwritten notes visible in the background include:

- $\log\left(\frac{a}{b}\right) = \log a - \log b$ (SMT)
- $a^{\log_e b} = b^{\log_e a}$
- $\sum_{r=1}^n (a+b)^r = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + \dots + {}^nC_n b^n$
- $\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$ (MJO)
- Perpendicular Lines $\Rightarrow m_1 m_2 = -1$
- $1 + e^{\frac{i\theta}{2}} = 2 \cos \frac{\theta}{2} e^{\frac{i\theta}{2}}$
- $\sin 2\theta = 2 \sin \theta \cos \theta$
- $y = \sin x$ graph with period 2π
- Equation of Rotation: $\frac{z_3 - z_1}{z_2 - z_1} = \frac{|z_3 - z_1|}{|z_2 - z_1|} e^{i\theta}$
- $|a| + |b| = |a - b|$ Solution: $\alpha\beta \leq 0$
- $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$
- $ax^2 + bx + c = 0$ Sum = $-\frac{b}{a}$, Product = $\frac{c}{a}$
- $y^2 = 4ax$
- $a = b = c$
- $Q! = 1$
- $a^2 - b^2 = (a-b)(a+b)$
- $\sqrt{x^2} = |x|$
- $\tan \theta$
- Newton's Formula: $ax^2 + bx + c = 0$, $S_n = p\alpha^n + q\beta^n$, $\Rightarrow aS_n + bS_{n-1} + cS_{n-2} = 0$
- $D_n = n! \left\{ \frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \dots \right\}$
- $\sin^2 \theta + \cos^2 \theta = 1$
- $\sec^2 \theta - \tan^2 \theta = 1$
- $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$
- $\sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{6}$
- Little Golu kya hai bolu!!
- PA = PB
- PACB is cyclic Quad
- $z^3 = 1 \Rightarrow z = 1, \omega, \omega^2$
- $1 + \omega + \omega^2 = 0$ & $\omega^3 = 1$
- $\bar{\omega} = \omega^2$ & $\bar{\omega}^2 = \omega$

Physics Wallah

Sachin Jakhar
(Sacchhu Sir)

Ab BACKLOG ko Kaho BBye!!

Complete Theory | Basic to Advanced Problems | Special Tips & Tricks
PYQ's Main & Advanced | Detailed Solutions | Pure Classroom Feel

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BASIC POINTS TO NOTE

Note-01

Square root of positive real number is always positive.

$$\sqrt{x^2} = |x|$$

Note-02

Division by zero is not-defined.

Note-03

Even root of only non-negative real numbers is defined. Odd root of every real number is defined.

Note-04

Multiplication and division (cancellation) is only valid for non-zero numbers/expressions.

(काटा-पीटी-Non zero न. ki होती है)

Note-05

Square (or even powers) of any number or expression is always +ve or can be zero.

Question

Q. If a, b, c are real numbers satisfying $2a^2 + 5b^2 + 9c^2 - 4ab + 2b - 6ac + 1 = 0$, then $a + b + c = \underline{\hspace{2cm}}$.

Sol. $2a^2 + 5b^2 + 9c^2 - 4ab + 2b - 6ac + 1 = 0$
 $\Rightarrow a^2 + 4b^2 - 2(a)(2b) + 9c^2 + a^2 - 2(a)(3c) + b^2 + 2b(1) + 1^2 = 0$
 $\Rightarrow (a - 2b)^2 + (3c - a)^2 + (b + 1)^2 = 0$

$$\Rightarrow a = 2b, 3c = a, b = -1$$

$$\Rightarrow a = -2, b = -1, c = \frac{-2}{3}$$

$$\Rightarrow a + b + c = \frac{-11}{3} \text{ fnf.}$$

IDENTITIES

एक ही नारा हमारा,
रट डाल ओ यारा

Part-01

हम जानते हैं! बचपन का ♥

$$(1) (a + b)^2 = a^2 + 2ab + b^2 = (a - b)^2 + 4ab$$

$$(2) (a - b)^2 = a^2 - 2ab + b^2 = (a + b)^2 - 4ab$$

$$(3) a^2 - b^2 = (a - b)(a + b)$$

$$a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$$

$$(4) (a + b)^3 = a^3 + b^3 + 3ab(a + b) = a^3 + b^3 + 3a^2b + 3ab^2$$

$$(5) (a - b)^3 = a^3 - b^3 - 3ab(a - b) = a^3 - b^3 - 3a^2b + 3ab^2$$

$$(6) a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$$

$$(7) a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$$

Part-02

Thodi stylish wali

$$(1) (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$(2) (a + b + c + d)^2 = (\text{Sabhi ke squares ka sum})$$

$$+ 2(\text{sum of multiplication of every two})$$

$$= (a^2 + b^2 + c^2 + d^2)$$

$$+ 2(ab + ac + ad + bc + bd + cd)$$

Part-03

(Impressive)

$$(1) a^2 + b^2 + c^2 - ab - bc - ca$$

$$= \frac{(a - b)^2 + (b - c)^2 + (c - a)^2}{2}$$

$$(2) a^3 + b^3 + c^3 - 3abc = (a + b + c)$$

$$(a^2 + b^2 + c^2 - ab - bc - ca)$$

Note

If $a^3 + b^3 + c^3 = 3abc$

or $\frac{a^3 + b^3 + c^3}{abc} = 3$

$\Rightarrow (a + b + c = 0) \text{ or } (a = b = c)$

Question

Q. The value of

$$\frac{(1.5)^3 + (4.7)^3 + (3.8)^3 - 3 \times 1.5 \times 4.7 \times 3.8}{(1.5)^2 + (4.7)^2 + (3.8)^2 - 1.5 \times 4.7 - 1.5 \times 3.8 - 4.7 \times 3.8}$$

is

(A) 8

(B) 9

(C) 10

(D) 11

Sol.

$$\frac{(1.5)^3 + (4.7)^3 + (3.8)^3 - 3 \times 1.5 \times 4.7 \times 3.8}{(1.5)^2 + (4.7)^2 + (3.8)^2 - 1.5 \times 4.7 - 1.5 \times 3.8 - 4.7 \times 3.8}$$

say: $1.5 = a, 4.7 = b, 3.8 = c$

$$\begin{aligned} & \frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca} \\ &= \frac{(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)}{(a^2 + b^2 + c^2 - ab - bc - ca)} \\ &= a + b + c = 1.5 + 4.7 + 3.8 = 10 \text{ fnf.} \end{aligned}$$

Question [JEE Main-2021]

Q. If $a + b + c = 1$, $ab + bc + ca = 2$ and $abc = 3$ then the value of $a^4 + b^4 + c^4$ is equal to

Sol. $a + b + c = 1$

SBS: $a^2 + b^2 + c^2 + 2(ab + bc + ca) = 1$

$\Rightarrow a^2 + b^2 + c^2 = -3$

$ab + bc + ca = 2$

SBS: $(ab)^2 + (bc)^2 + (ca)^2 + 2(ab^2c + abc^2 + a^2bc) = 4$

$\Rightarrow a^2b^2 + b^2c^2 + c^2a^2 + 2(abc)(b + c + a) = 4$

$\Rightarrow a^2b^2 + b^2c^2 + c^2a^2 = -2$

$a^2 + b^2 + c^2 = -3$

SBS: $(a^2)^2 + (b^2)^2 + (c^2)^2 + 2(a^2b^2 + b^2c^2 + c^2a^2) = 9$

$\Rightarrow a^4 + b^4 + c^4 + 2(-2) = 9$

$\Rightarrow a^4 + b^4 + c^4 = 13 \text{ fnf.}$

Module Question

Q. If $a, b, c \in \mathbb{R}$ and $a, b, c \neq 0$ such that

$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 6$ and $\frac{b}{a} + \frac{c}{b} + \frac{a}{c} = 8$, then

$\frac{a^3}{b^3} + \frac{b^3}{c^3} + \frac{c^3}{a^3} - 3$ is equal to

(A) 81

(B) 48

(C) 72

(D) 84

Sol. Put: $\frac{a}{b} = x, \frac{b}{c} = y, \frac{c}{a} = z$

$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 6 \Rightarrow x + y + z = 6 \quad \dots(1)$

$\frac{b}{a} + \frac{c}{b} + \frac{a}{c} = 8 \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 8 \quad \dots(2)$

and $xyz = 1 \quad \dots(3)$

SBS: eqⁿ (1)

$x^2 + y^2 + z^2 + 2(xy + yz + zx) = 36$

$x^2 + y^2 + z^2 = 36 - 16 = 20$

Now, $\frac{a^3}{b^3} + \frac{b^3}{c^3} + \frac{c^3}{a^3} - 3 \quad (1)$

$= x^3 + y^3 + z^3 - 3(xyz)$

$= (x + y + z)(x^2 + y^2 + z^2 - (xy + yz + zx))$

$= 6(20 - (8)) = 6 \times 12 = 72 \text{ fnf}$

Part-04

420 wali (चार सौ बीस वाली) ★★ ★

$a^4 + a^2 + 1 = (a^2 - a + 1)(a^2 + a + 1)$

$(a^2)^2 + (1)^2 + 2a^2 - a^2$

$= (a^2 + 1)^2 - (a)^2$

$= (a^2 + 1 + a)(a^2 + 1 - a)$

Part-05

Term & uska Reciprocal

$\left(x + \frac{1}{x}\right)^2 - 2 = x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$

Question [IIT JEE-1980]

Q. If $x > y > 0$, then show that the expression

$\left(\sqrt{2} \left(2x + \sqrt{x^2 - y^2}\right) \left(\sqrt{x - \sqrt{x^2 - y^2}}\right)\right)$ can be

simplified to $(\sqrt{x + y})^3 - (\sqrt{x - y})^3$

Sol. Say: $x + y = a$ & $x - y = b$
 $\Rightarrow 2x = a + b, 2y = a - b, x^2 - y^2 = ab$

$$\Rightarrow 2x + \sqrt{x^2 - y^2} = a + b + \sqrt{ab}$$

$$\text{and } \sqrt{x - \sqrt{x^2 - y^2}} = \sqrt{\frac{a+b}{2} - \sqrt{ab}}$$

$$= \sqrt{\frac{(\sqrt{a})^2 + (\sqrt{b})^2 - 2\sqrt{ab}}{2}}$$

Required expression

$$= \sqrt{2} \left(a + b + \sqrt{ab} \right) \frac{(\sqrt{a} - \sqrt{b})}{\sqrt{2}}$$

$$= (\sqrt{a})^3 - (\sqrt{b})^3 = (\sqrt{x+y})^3 - (\sqrt{x-y})^3 \quad \text{HHPP}$$

DIFFERENT TYPES OF NUMBERS

Number system (Informative)

Natural numbers: $(N) = \{1, 2, 3, 4, 5, \dots\}$

Whole numbers: $(W) = \{0, 1, 2, 3, 4, 5, \dots\}$

Integers: $(I/Z) = \{\dots, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots\}$

Prime Numbers: Natural numbers which are divisible by 1 and itself only are called prime numbers,

Ex. $\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, \dots\}$

Composite Numbers: Let 'a' be a natural number, 'a' is said to be composite if, it has atleast three distinct factors.

Ex. $\{4, 6, 8, 9, 10, 12, \dots\}$

Co-prime Numbers: Two natural numbers (not necessarily prime) are called coprime, if their H.C.F. (Highest common factor) is one.

These numbers are also called **relatively prime numbers**.

Co-prime
or
relative prime

(a, b)

$HCF(a, b) = 1$

or

a & b में No
common factor
(except 1)

नाम में Prime है, लेकिन
numbers ka prime होना
जरूरी नहीं

Ex. $(2, 3); (3, 5); (1, 6); (8, 9)$

Twin prime Numbers: If the difference between two prime numbers is two, then the numbers are called twin prime numbers.

Twin prime No
ऐसे दो Prime no.'s
जिनका difference
'2' हो

ex: $(2, 3) \times$
 $(3, 5) \checkmark$
 $(5, 7) \checkmark$
 $(1, 3) \times$
 $(11, 13) \checkmark$

Rational Numbers:

$$Q = \frac{p}{q}, (p \text{ \& } q \rightarrow \text{Integer}) q \neq 0$$

$$Q = \left\{ -\frac{1}{2}, 5, -\frac{3}{7}, \frac{1}{8}, \frac{11}{13}, \frac{6}{4}, 1, 2, 3, 4, 5, 0, -1, -2, -3 \right\}$$

Integers

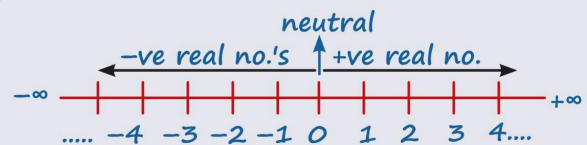
Irrational Number:

$$\bar{Q} / Q^c = \{ \sqrt{2}, \sqrt{3}, -\sqrt{5}, \sqrt{7}, \sqrt{11}, \sqrt{8}, \sqrt{14}, \dots, \pi, e, \dots \}$$

Real Number: $R = Q + \bar{Q}$

rational irrational

Number Line:



INTERVALS & SOLUTION REPRESENTATION

● Dark Circle $\rightarrow$ jisko include karna ho

○ Hollow Circle $\rightarrow$ jisko exclude karna ho

(,) Open bracket $\rightarrow$ Jisko exclude karna ho

[,] Closed bracket $\rightarrow$ Jisko include karna ho

{,} Curly bracket $\rightarrow$ discrete value ke liye {kuch value ke liye}

$\cup$ Union $\rightarrow A \cup B \equiv$ combined value of A & B.

$\cap$ Intersection $\rightarrow A \cap B \equiv$ common value of A & B.

$\in$ belongs to $\rightarrow$ If 'a' set M में present है

$\Rightarrow a \in M$

Q. Match the column:

Column-I		Column-II		Column-III	
(A)	$-2 < x \leq 3, x \in \mathbb{R}$	(P)		(x)	$x \in [0, 4) - \{2, 3\}$
(B)	$0 \leq x < 4 \& x \neq 2, 3, x \in \mathbb{R}$	(Q)		(y)	$x \in [-4, 5] \cup (7, \infty)$
(C)	$(-5 \leq x < 0) \text{ or } (x=3), x \in \mathbb{R}$	(R)		(z)	$x \in [-5, 0) \cup \{3\}$
(D)	$(-4 \leq x \leq 5) \text{ or } (x > 7), x \in \mathbb{R}$	(S)		(T)	$x \in (-2, 3]$

RULES OF INEQUALITY

Rule-01

We can add (or subtract) any number 'k' on both sides of inequality. Doing this will not change the sign of inequality.

Rule-02

We can multiply (or divide) any non-zero number 'k' on both sides of inequality and sign of inequality will change according to sign of 'k' as:

If $k > 0$, then

sign of inequality will remains same,

If $k < 0$, then

sign of inequality will get reversed.

WAVY CURVE METHOD

Step-01: Check structure:

$$\frac{N^r}{D^r} > 0 \rightarrow \text{ek taraf zero}$$

Step-02: Find critical points (CP's)

Step-03: Plot all CP's on number line

Step-04: Check sign of given expression by putting value of x from different parts of number line.

Step-05: Jo pucha gya, use dekh key jawab do.

BAAT MAIN DUM HAI

1. Roots of denominator "always" → hollow circle → always excluded.

2. In case of $\geq, \leq$ roots of numerator → Roots of N^r पर dark circle → included

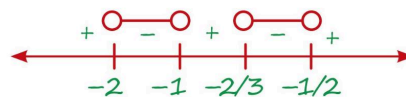
3. In case of $>, <$ roots → सभी पर Hollow circle

Question [IIT JEE-1987]

Q. Find the set of all x for which

$$\frac{2x}{2x^2 + 5x + 2} > \frac{1}{x+1}$$

$$\begin{aligned} \text{Sol. } \frac{2x}{2x^2 + 5x + 2} - \frac{1}{x+1} &> 0 \\ \frac{2x^2 + 2x - 2x^2 - 5x - 2}{(2x^2 + 5x + 2)(x+1)} &> 0 \\ \frac{(3x+2)^1}{(x+2)^1(2x+1)^1(x+1)^1} &< 0 \end{aligned}$$



$$x \in (-2, -1) \cup \left(-\frac{2}{3}, -\frac{1}{2}\right) \text{ fnf.}$$

Acidic Module Question

Q. Number of non-negative integral value of x satisfying the inequality

$$\frac{2}{x^2 - x + 1} - \frac{1}{x+1} - \frac{2x-1}{x^3+1} \geq 0 \text{ is}$$

(A) 0 (B) 1 (C) 2 (D) 3

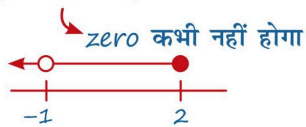
Sol. $\left(\frac{2}{x^2 - x + 1} - \frac{1}{x + 1}\right) - \frac{2x - 1}{x^3 + 1} \geq 0$

$$\Rightarrow \left(\frac{2x + 2 - x^2 + x - 1}{(x + 1)(x^2 - x + 1)}\right) - \frac{2x - 1}{x^3 + 1} \geq 0$$

$$\Rightarrow \frac{3x - x^2 + 1 - 2x + 1}{(x^3 + 1)} \geq 0$$

$$\Rightarrow \frac{x - x^2 + 2}{(x^3 + 1)} \geq 0 \Rightarrow \frac{x^2 - x - 2}{x^3 + 1} \leq 0$$

$$\Rightarrow \frac{(x + 1)(x - 2)}{(x + 1)(x^2 - x + 1)} \leq 0$$



$x \in (-\infty, -1) \cup (-1, 2]$ or
 $x \in (-\infty, 2] - \{-1\}$ fnf.

TRICK



1. Make coefficient 'x', positive in each linear factor.
2. Now start marking (+/-/+/-/+/-) from right to left.
3. Remember:

Jinn factors ki net power odd hai, uske about $\rightarrow$ sign change hota hai.
 Jinn factors ki net power even hai, uske about $\rightarrow$ sign change nahi hota

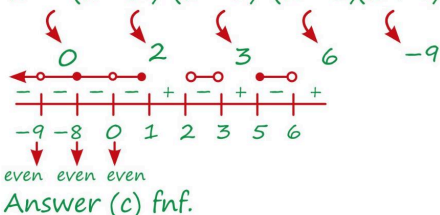
Question

Q. The complete solution set of inequality

$$\frac{(x - 5)^{1005}(x + 8)^{1008}(x - 1)}{x^{1006}(x - 2)^3(x - 3)^5(x - 6)(x + 9)^{1010}} \leq 0 \text{ is}$$

- (A) $(-\infty, -9) \cup (-8, 0) \cup (0, 1) \cup (2, 3) \cup [5, 6)$
 (B) $(-\infty, -9) \cup (-9, 0) \cup (0, 1) \cup (2, 3) \cup (5, 6)$
 (C) $(-\infty, -9) \cup (-9, 0) \cup (0, 1] \cup (2, 3) \cup [5, 6)$
 (D) $(-\infty, 0) \cup (0, 1] \cup (2, 3) \cup [5, 6)$

Sol. $\frac{(x - 5)^{1005}(x + 8)^{1008}(x - 1)}{x^{1006}(x - 2)^3(x - 3)^5(x - 6)(x + 9)^{1010}} \leq 0$



DOUBLE INEQUALITY

$$h(x) < f(x) \leq g(x)$$

$$h(x) < f(x) \cap f(x) \leq g(x)$$

Question

Q. The number of the integral solutions of

$$x^2 + 9 < (x + 3)^2 < 8x + 25 \text{ is}$$

- (A) 1 (B) 2
 (C) 4 (D) 5

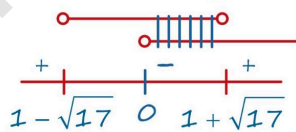
Sol.

$$x^2 + 9 < (x + 3)^2 < 8x + 25$$

$$x^2 + 9 < x^2 + 9 + 6x \Rightarrow 0 < 6x \Rightarrow x > 0$$

$$x^2 + 9 + 6x < 8x + 25 \Rightarrow x^2 - 2x - 16 < 0$$

$$(x - (1 + \sqrt{17}))(x - (1 - \sqrt{17})) < 0$$



$$x \in (0, 1 + \sqrt{17}) \equiv (0, 5. \text{ something})$$

Integers $\Rightarrow x = 1, 2, 3, 4, 5$

ROLE OF QUADRATIC

$$D = b^2 - 4ac$$

$$D \geq 0 \rightarrow \text{factorisable}$$

$$D < 0 \rightarrow \text{non-factorisable}$$

Type: Non-factorisable quadratic

$$\text{Quadratic} \equiv (ax^2 + bx + c)$$

$$\Rightarrow \text{If } a > 0 \text{ and } D < 0 \Rightarrow \text{हमेशा +ve होगी}$$

$$\Rightarrow \text{If } a < 0 \text{ and } D < 0 \Rightarrow \text{हमेशा -ve होगी}$$

Q. If $f(x) = \frac{(x^2 - x + 6)(x - 5)}{(x + 2)(3x - 8 - x^2)}$ and solution

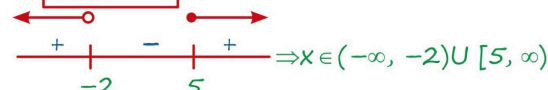
set of $f(x) \leq 0$ is $x \in (-\infty, -a) \cup [b, \infty)$ where $a, b \in \mathbb{N}$, then value of $\text{GCD}(2a^4 + 4, b^2 - 1)$ is _____.

Sol.

$$a > 0, D < 0 \Rightarrow +ve \text{ always}$$

$$\Rightarrow f(x) = \frac{(x^2 - x + 6)(x - 5)}{(x + 2)(3x - 8 - x^2)} \leq 0$$

$$a < 0, D < 0 \Rightarrow -ve \text{ always}$$

$$\Rightarrow \frac{x - 5}{x + 2} \geq 0$$


$$\Rightarrow x \in (-\infty, -2) \cup [5, \infty)$$

$a = 2, b = 5$
 $GCD(2a^4 + 4, b^2 - 1)$
 $HCF(36, 24) = 12 \text{ fnf}$

Question [IIT JEE 1982] ★★ ★ New to me

Q. The largest interval for which

$$x^{12} - x^9 + x^4 - x + 1 > 0 \text{ is}$$

- (A) $-4 < x \leq 0$ (B) $0 < x < 1$
 (C) $-100 < x < 100$ (D) $-\infty < x < \infty$

Sol. $x \geq 1 \Rightarrow x^{12} - x^9 + x^4 - x + 1 > 0 \rightarrow (+ve)$
 $x \leq 0 \Rightarrow (x^{12} + x^4 + 1) - (x^9 + x) \rightarrow (+ve)$
 $0 < x < 1 \Rightarrow 1 - x + x^4 - x^9 + x^{12} \rightarrow (+ve)$
 $\Rightarrow$ Hence $x \in \mathbb{R}$ or $(-\infty < x < \infty)$ fnf.

EQUATIONS CONTAINING SQUARE ROOT

Irrational equations $\rightarrow$ Power is in fraction

Squaring करने से Extra roots आ सकते हैं।

Verify $\rightarrow$ all values obtained after squaring

Q. Solve: $\sqrt{6 - 4x - x^2} = x + 4$

Sol. $\sqrt{6 - 4x - x^2} = x + 4$
 SBS $\Rightarrow 6 - 4x - x^2 = (x + 4)^2$
 $\Rightarrow 6 - 4x - x^2 = x^2 + 16 + 8x$
 $\Rightarrow 0 = 2(x^2 + 6x + 5)$
 $\Rightarrow 0 = 2(x + 5)(x + 1)$

$x = -5, -1$, now check these values

$$\sqrt{6 - 4x - x^2} = x + 4$$

$$\sqrt{6 - 4(-5) - (-5)^2} = -5 + 4 \Rightarrow 1 = -1 \quad \times$$

$x = -5$ $\leftarrow$ Reject (extra root)

$$\sqrt{6 - 4x - x^2} = x + 4$$

$$\sqrt{6 - 4(-1) - (-1)^2} = -1 + 4 \Rightarrow 3 = 3 \quad \checkmark$$

$x = -1$ $\leftarrow$ Accepted & fnf.

Question [IIT-1997]

Q. The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has

- (A) no solution (B) one solution
 (C) two solution (D) more than 2 sol.

Sol. $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$
 SBS: $(x+1) + (x-1) - 2\sqrt{x+1}\sqrt{x-1} = 4x-1$
 $\Rightarrow -2\sqrt{x+1}\sqrt{x-1} = 2x-1$
 SBS: $4(x+1)(x-1) = 4x^2 + 1 - 4x$
 $\Rightarrow (4x^2 - 4) = 4x^2 - 4x + 1$
 $\Rightarrow 4x = 5 \Rightarrow x = \frac{5}{4}$ (Rejected) $\Rightarrow$ No sol.

Question [JEE Main-2023]

Q. The number of real roots of the equation

$$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}, \text{ is}$$

- (A) 0 (B) 1 (C) 3 (D) 2

Sol. $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$
 $\sqrt{(x-3)(x-1)} + \sqrt{(x-3)(x+3)} - \sqrt{(x-3)(4x-2)} = 0$

$$\sqrt{x-3}(\sqrt{x-1} + \sqrt{x+3} - \sqrt{4x-2}) = 0$$

$$\sqrt{x-3} = 0 \rightarrow x = 3$$

$$\sqrt{x-1} + \sqrt{x+3} = \sqrt{4x-2}$$

$$\text{SBS: } x-1 + x+3 + 2\sqrt{x-1}\sqrt{x+3} = 4x-2$$

$$\Rightarrow 2\sqrt{x-1}\sqrt{x+3} = 2(x-2)$$

$$\text{SBS: } (x-1)(x+3) = x^2 + 4 - 4x$$

$$\Rightarrow x^2 + 2x - 3 = x^2 + 4 - 4x$$

$$\Rightarrow 6x = 7 \Rightarrow x = \frac{7}{6} \text{ (rejected.)}$$

Hence only one root $x = 3$ fnf.

IRRATIONAL INEQUALITIES

Another rules of inequality

Squaring both side in an inequality is only allowed when both sides are non-negative (+ve or zero)

Q. $\sqrt{x-2} \geq \sqrt{5-x}$

Sol. $\sqrt{x-2} \geq \sqrt{5-x}$

Defining

$$\begin{cases} x-2 \geq 0 \Rightarrow x \geq 2 \\ 5-x \geq 0 \Rightarrow 5 \geq x \end{cases}$$

Solution SBS:

$$x-2 \geq 5-x$$

$$2x \geq 7 \Rightarrow x \geq \frac{7}{2}$$

fnc = defining $\cap$ solution

$x \in [2, 5]$

$x \in [\frac{7}{2}, 5]$

Q. Solve: $2 - \frac{2x}{3} \geq \sqrt{x+4}$

Sol. **Sumit:** Defining: $x+4 \geq 0 \Rightarrow x \geq -4$

Case-I: Where LHS is +ve/0

$$\Rightarrow 2 - \frac{2x}{3} \geq 0 \Rightarrow 3 \geq x \text{ (initial condition)}$$

$$\Rightarrow 2 - \frac{2x}{3} \geq \sqrt{x+4}$$

$$\Rightarrow \text{SBS: } \frac{4}{9}(9+x^2-6x) \geq x+4$$

$$\Rightarrow x(4x-33) \geq 0$$

$C-I: x \in (-\infty, 0]$

Case-II: When LHS is -ve

$$\underbrace{2 - \frac{2x}{3}}_{-ve} \geq \underbrace{\sqrt{x+4}}_{+ve} \Rightarrow \emptyset$$

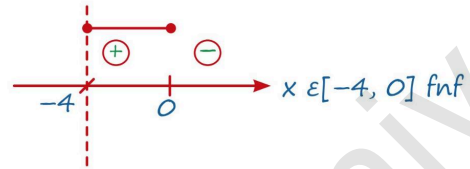
fnc = Defining $\cap$ solution

$x \in [-4, 0]$ fnc

Santosh: Defining $x+4 \geq 0 \Rightarrow x \geq -4$

$$\Rightarrow \text{WCM} \Rightarrow 2 - \frac{2x}{3} - \sqrt{x+4} \geq 0$$

As, Roots of LHS is $x = 0$



MODULUS FUNCTION

Also called as Absolute value function

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Ex: $|x-5| = x-5, \quad x-5 \geq 0$
 $= -(x-5), \quad x-5 < 0$

Ex: $|(x^2 - 5x + 6)| = x^2 - 5x + 6, \quad x \leq 2$
 $= -(x^2 - 5x + 6), \quad 2 < x < 3$
 $= x^2 - 5x + 6, \quad x \geq 3$

PROPERTIES OF MOD

- जब भी modulus aaye to sabse pehle use हटाओ
- $\sqrt{x^2} = |x|$
- $|x^2| = |x|^2 = x^2$
- $|ab| = |a||b|$
- $|a^n| = |a|^n$
- $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
- $|a \pm b| \neq |a| \pm |b|$ (in general)

MODULUS EQUATION

- If $\alpha > 0$ and $|x| = \alpha$, then $x = \pm \alpha$
- If $\alpha \in \mathbb{R}$ and $|x| = |\alpha|$, then $x = \pm \alpha$



Question [JEE Main-2023]

Q. The sum of all the roots of the equation $|x^2 - 8x + 15| - 2x + 7 = 0$ is

- (A) $9 + \sqrt{3}$ (B) $11 + \sqrt{3}$
(C) $9 - \sqrt{3}$ (D) $11 - \sqrt{3}$

Sol. Sunita:

$$|x^2 - 8x + 15| = 2x - 7$$

$$\begin{array}{l} \oplus \Rightarrow x^2 - 8x + 15 = 2x - 7 \\ \Rightarrow x^2 - 10x + 22 = 0 \\ \Rightarrow x = 5 \pm \sqrt{3} \\ \ominus \Rightarrow x^2 - 8x + 15 = -(2x - 7) \\ \Rightarrow x^2 - 6x + 8 = 0 \\ \Rightarrow x = 2, 4 \end{array}$$

For $|x| = \alpha \Rightarrow \alpha \geq 0$

$$2x - 7 \geq 0 \Rightarrow x \geq \frac{7}{2}$$

$$x = 2, 4, 5 + \sqrt{3}, 5 - \sqrt{3} \Rightarrow \text{sum} = 9 + \sqrt{3} \text{ fnf}$$

Sushmita: $|(x - 5)(x - 3)| - 2x + 7 = 0$

Mod हटाओ

C-I: $x \in (-\infty, 3] \cup [5, \infty)$

$$\begin{array}{c} + \quad - \quad + \\ \hline 3 \quad 5 \end{array}$$

$$\Rightarrow x^2 - 8x + 15 - 2x + 7 = 0$$

$$x^2 - 10x + 22 = 0$$

$$x = 5 \pm \sqrt{3}$$

$\Rightarrow$ only $x = 5 + \sqrt{3}$ is accepted

$$x \in (-\infty, 3] \cup [5, \infty)$$

C-II: Now, for $x \in (3, 5)$

$$-(x^2 - 8x + 15) - 2x + 7 = 0$$

$$\Rightarrow -x^2 + 6x - 8 = 0$$

$$\Rightarrow x = 2, 4$$

$\Rightarrow$ only $x = 4$ is accepted.

Question [JEE Adv.-2021]

Q. For $x \in \mathbb{R}$, the number of real roots of the equation: $3x^2 - 4|x^2 - 1| + x - 1 = 0$ is

Sol.

$$\begin{array}{c} + \quad - \quad + \\ \hline -1 \quad 1 \end{array}$$

$$3x^2 - 4|(x - 1)(x + 1)| + x - 1 = 0$$

Case-I: $x \in (-\infty, -1] \cup [1, \infty)$ (initial condⁿ.)

$$\Rightarrow 3x^2 - 4(x^2 - 1) + x - 1 = 0$$

$$\Rightarrow x^2 - x - 3 = 0$$

$$x = \frac{1 \pm \sqrt{1 - 4(-3)}}{2} = \frac{1 \pm \sqrt{13}}{2} \text{ (both accepted)}$$

Case-II: $x \in (-1, 1)$ (initial condition)

$$3x^2 + 4(x^2 - 1) + x - 1 = 0$$

$$\Rightarrow 7x^2 + x - 5 = 0$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 - 4(7)(-5)}}{2(7)} = \frac{1 \pm \sqrt{141}}{14}$$

(both are accepted)

$\Rightarrow$ Number of real roots = 4 fnf.

Question [JEE Main-2024]

Q. The number of real solutions of the equation $x|x + 5| + 2|x + 7| - 2 = 0$ is

Sol. $\begin{array}{c} x < -7 \quad -7 \leq x < -5 \quad x \geq -5 \\ \hline -7 \quad -5 \end{array}$

Case-I: $x < -7$ (initial condition)

$$x(-x - 5) + 2(-x - 7) - 2 = 0$$

$$\Rightarrow -x^2 - 5x - 2x - 14 - 2 = 0$$

$$\Rightarrow x^2 + 7x + 16 = 0 \Rightarrow x \in \emptyset \text{ (No real roots)}$$

Case-II: $-7 \leq x < -5$ (initial condition)

$$-x(x + 5) + 2(x + 7) - 2 = 0$$

$$\Rightarrow -x^2 - 5x + 2x + 14 - 2 = 0$$

$$\Rightarrow x^2 - 3x - 12 = 0$$

$$\Rightarrow x = \frac{-3 \pm \sqrt{57}}{2} \begin{array}{l} \ominus \rightarrow -5. \text{ something } (\sqrt{v}) \\ \oplus \rightarrow 2. \text{ something } (x) \end{array}$$

Case-III: $x \geq -5$ (initial condition)

$$x(x + 5) + 2(x + 7) - 2 = 0$$

$$\Rightarrow x^2 + 5x + 2x + 14 - 2 = 0$$

$$\Rightarrow x^2 + 7x + 12 = 0$$

$$\Rightarrow (x + 3)(x + 4) = 0$$

$$\Rightarrow x = -3, -4 \text{ (both accepted)}$$

$\Rightarrow$ Number of real roots = 3 fnf.

Q. Number of solution equation

$$|5x + 2| + |3 - x| = 7x + 5$$

- (A) 0 (B) 1
(C) 2 (D) 3

Sol. Case-I: $x < -2/5$

$$-(5x + 2) + 3 - x = 7x + 5$$

$$\Rightarrow -5x - 2 + 3 - x = 7x + 5$$

$$\Rightarrow 1 - 5 = 13x \Rightarrow \frac{-4}{13} = x \text{ Rejected}$$

Case-II: $-2/5 \leq x < 3$

$$(5x+2)+(3-x) = 7x+5$$

$$\Rightarrow 4x+5 = 7x+5$$

$x=0$ Accepted

Case-III: $x \geq 3$

$$5x+2+(-(3-x)) = 7x+5$$

$$\Rightarrow 5x+2-3+x = 7x+5$$

$x=-6$ Rejected

$\Rightarrow$ No. of sol = 1 fnf.

MODULUS INEQUALITY

100%
Question
aayega!!

KKPNCC

1. If $|x| < \alpha \Rightarrow -\alpha < x < \alpha$
2. If $|x| \leq \alpha \Rightarrow -\alpha \leq x \leq \alpha$
3. If $|x| > \alpha \Rightarrow x < -\alpha$ union $x > \alpha$
4. If $|x| \geq \alpha \Rightarrow x \leq -\alpha$ union $x \geq \alpha$

Question

Q. If solution set of inequality $|x-2| - 3 \leq 2$ is $x \in [-a, b] \cup [a, c]$ where $b < a < c$, then $a + b + c$ is _____.

Sol. Say: $|x-2| - 3 = \alpha$

$$\Rightarrow |\alpha| \leq 2 \Rightarrow -2 \leq \alpha \leq 2$$

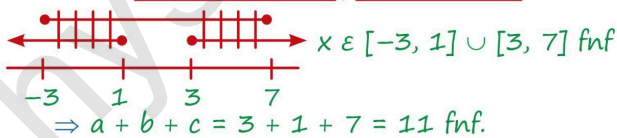
$$-2 \leq |x-2| - 3 \leq 2$$

$$1 \leq |x-2| \leq 5$$

$$1 \leq |x-2| \Rightarrow |x-2| \leq 5$$

$$(x-2 \leq -1) \cup (1 \leq x-2) \Rightarrow -5 \leq x-2 \leq 5$$

$$x \leq 1 \cup 3 \leq x \quad -3 \leq x \leq 7$$



Question [JEE Main-2022]

Q. Let $S = \{x \in [-6, 3] : \frac{|x+3|-1}{|x|-2} \geq 0\}$

and $T = \{x \in \mathbb{Z} : x^2 - 7|x| + 9 \leq 0\}$. Then the number of elements of $S \cap T$ is

(A) 7 (B) 5 (C) 4 (D) 3

Sol. $\frac{|x+3|-1}{|x|-2} \geq 0$

$x = -2, -4$

$x = 2, -2$

$$S \Rightarrow x \in [-6, -4] \cup (2, 3]$$

integers are $x = -6, -5, -4, 3$

Check these 4 integers in T

$$\Rightarrow x^2 - 7|x| + 9 \leq 0$$

$$\Rightarrow x = -6$$

$$36 - 7 \times 6 + 9 = 3 \leq 0 \times$$

$$\Rightarrow x = -5$$

$$25 - 35 + 9 = -1 \leq 0 \checkmark$$

$$\Rightarrow x = -4$$

$$16 - 28 + 9 = -3 \leq 0 \checkmark$$

$$\Rightarrow x = 3$$

$$9 - 21 + 9 = -3 \leq 0 \checkmark$$

$\Rightarrow$ No. of element in $(S \cap T) = 3$ fnf.

SPECIAL RESULT

★★★

IMPORTANT FORMAT

1. $|\alpha| + |\beta| = |\alpha + \beta| \Rightarrow$ solution: $\alpha\beta \geq 0$
2. $|\alpha| + |\beta| > |\alpha + \beta| \Rightarrow$ solution: $\alpha\beta < 0$
3. $|\alpha| + |\beta| = |\alpha - \beta| \Rightarrow$ solution: $\alpha\beta \leq 0$
4. $|\alpha| + |\beta| > |\alpha - \beta| \Rightarrow$ solution: $\alpha\beta > 0$

Blue बादल (KKPNCC)

अगर same sign वाली satisfy करे तो $\Rightarrow \alpha\beta > 0$

अगर opposite sign वाली satisfy करे तो $\Rightarrow \alpha\beta < 0$
equality ($\geq, \leq$) के लिए किसी एक α or β को zero put करके देखो

Blue बादल (KKPNCC)

Hint: ek traf दो अलग-अलग mod & ek side ek hi mod.

Q. Solve for values of x:

$$(A) |3x-5| + |8-x| = |3+2x|$$

$$(B) |x^2-5x+6| + |x^2-4| = 5|x-2|$$

Sol. (A) $\frac{\alpha}{|3x-5|} + \frac{\beta}{|8-x|} = \frac{\alpha+\beta}{|3+2x|}$

$$|\alpha| + |\beta| = |\alpha + \beta| \Rightarrow \alpha\beta \geq 0$$

$$(3x - 5)(8 - x) \geq 0$$

$$(3x - 5)(x - 8) \leq 0$$

$$\begin{array}{c} + \quad - \quad + \\ | \quad | \quad | \\ 5/3 \quad 8 \end{array} \Rightarrow x \in \left[\frac{5}{3}, 8 \right] \text{ fnf}$$

$$(B) \quad \overbrace{|x^2 - 5x + 6|}^{\alpha} + \overbrace{|x^2 - 4|}^{\beta} = \overbrace{5|x - 2|}^{|\beta - \alpha|}$$

$$\boxed{|\alpha| + |\beta| = |\beta - \alpha|}$$

$$\Rightarrow \alpha\beta \leq 0$$

$$(x^2 - 5x + 6)(x^2 - 4) \leq 0 \quad \text{even}$$

$$\Rightarrow (x - 2)^2(x - 3)(x + 2) \leq 0$$

$$x \in [-2, 3] \text{ fnf.}$$

Q. Solve: $|(x^2 + 5x + 9)| < |x^2 + 2x + 2| + |3x + 7|$.

Sol. $\begin{array}{c} \alpha \quad \beta \quad \alpha \quad \beta \\ |x^2 + 2x + 2 + 3x + 7| < |x^2 + 2x + 2| + |3x + 7| \\ |\alpha + \beta| < |\alpha| + |\beta| \Rightarrow \alpha\beta < 0 \\ (x^2 + 2x + 2)(3x + 7) < 0 \end{array}$

+ve always

$$\begin{array}{c} - \quad + \\ | \quad | \\ -7/3 \end{array} \quad \boxed{x \in \left(-\infty, -\frac{7}{3} \right)} \text{ fnf.}$$

Question [JEE Main-2021]

Q. If $A = \{x \in \mathbb{R}; |x - 2| > 1\}$, $B = \{x \in \mathbb{R}; \sqrt{x^2 - 3} > 1\}$, $C = \{x \in \mathbb{R}; |x - 4| \geq 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap Z$ is

Sol. Set A: $|x - 2| > 1$

$$\begin{array}{l} x - 2 < -1 \cup x - 2 > 1 \\ x < 1 \cup x > 3 \end{array}$$

$$A: \boxed{x \in (-\infty, 1) \cup (3, \infty)}$$

Set B: $\sqrt{x^2 - 3} > 1$

Soln: SBS

$$\begin{array}{l} x^2 - 3 > 1 \\ x^2 - 4 > 0 \\ (x - 2)(x + 2) > 0 \end{array} \quad \begin{array}{l} \text{defining} \\ x^2 - 3 \geq 0 \\ (x - \sqrt{3})(x + \sqrt{3}) \geq 0 \end{array}$$

$$B: \boxed{x \in (-\infty, -2) \cup (2, \infty)}$$

$$\text{set } C: |x - 4| - 2 \geq 0$$

$$\begin{array}{c} + \quad - \quad + \\ | \quad | \quad | \\ -2 \quad 6 \end{array}$$

$$C: \boxed{x \in (-\infty, -2) \cup [6, \infty)}$$

$$A \cap B \cap C \Rightarrow \boxed{x \in (-\infty, -2) \cup [6, \infty)}$$

$$\text{Complement} \Rightarrow (A \cap B \cap C)^c \in [-2, 6)$$

$$[-2, 6) \cap \mathbb{Z} \rightarrow \text{no. of elem} = 8$$

$$\Rightarrow \text{number of subsets} = 2^8 = 256 \text{ fnf}$$

Question [JEE Main-2023]

Q. Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set $S = \{x + \lambda : x \text{ is an integer solution of } E\}$ is

Sol. E: $\underbrace{|x|^2 - 2|x|}_{\text{Integer}} + \underbrace{|\lambda - 3|}_{\text{Integer}} = 0, x \in \mathbb{Z}$

$$\Rightarrow |x|^2 - 2|x| + 1 + |\lambda - 3| - 1 = 0$$

$$\boxed{(|x| - 1)^2 + |\lambda - 3| = 1}$$

Case-I: $|x| - 1 = 0$ & $|\lambda - 3| = 1$

$$x = \pm 1 \quad \& \quad \lambda = 4 \text{ or } 2$$

Case-II: $|x| - 1 = 1$ & $|\lambda - 3| = 0$

$$x = \pm 2 \quad \& \quad \lambda = 3$$

$$\Rightarrow \text{Largest value of } x + \lambda = 5 \text{ fnf.}$$

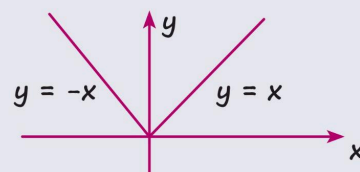
DOMAIN, RANGE & GRAPH OF MOD

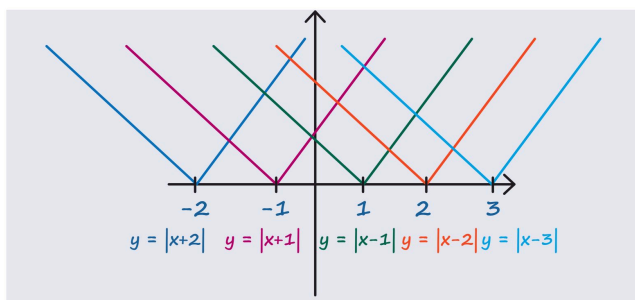
$$f(x) = |x|$$

$$\blacktriangle x \in \mathbb{R}$$

$$\blacktriangle \text{Range (value of mod)} \in [0, \infty)$$

Graph:





Blue बाबल (KKPNCC)

Use madam method (x ki value put kro, y nikalo, fir point बनाकर plot कर दो, Now join all points) to draw graph Mod of Linear function

Q. Match The Graph:

Column-I:		Column-II:	
(P)	$f(x) = x - 1 + x - 2 + x - 3 $	(A)	
(Q)	$f(x) = 3 x - 2 + x + 5 - 2 x $	(B)	
(R)	$f(x) = 2x - 3 - x + 5 $	(C)	
(S)	$f(x) = x - 3 - x + 4 $	(D)	

Sol. P-(C), Q-(B), R-(D), S-(A)

LOGARITHM



KKPNCC

Direct Question from log in Mains + Adv.

- $\log_B A = \log 'A' \text{ with base 'B'}$
- Value of $\log_B A = \text{base(B) ki power}$ में क्या रखें ki vo (A) बन जाए।
- $\log_B A$ is only defined when
 $\Rightarrow A > 0 \cap B > 0 \cap B \neq 1$ ★★ ★

Q. Values of x for which $f(x) = \log_{|x-3|} \left(\frac{|x|-5}{2-|x|} \right)$ is defined.

Sol. $f(x) = \log_{|x-3|} \left(\frac{|x|-5}{2-|x|} \right)$

$$\begin{aligned} & \left(\frac{|x|-5}{2-|x|} \right) > 0 \cap |x-3| > 0 \cap |x-3| \neq 1 \\ & \begin{array}{c} \text{---} \circ \text{---} \circ \text{---} \\ -5 \quad -2 \quad 2 \quad 5 \end{array} \cap \begin{array}{c} \text{---} \circ \text{---} \circ \text{---} \\ -5 \quad -2 \quad 2 \quad 5 \end{array} \cap \begin{array}{c} \text{---} \circ \text{---} \circ \text{---} \\ -5 \quad -2 \quad 2 \quad 5 \end{array} \\ & x \in \mathbb{R} - \{3\} \cap x - 3 \neq \pm 1 \Rightarrow x \neq 4, 2 \\ & \text{fnf} \Rightarrow x \in (-5, -2) \cup (2, 5) - \{3, 4\} \end{aligned}$$

PROPERTIES



Sara khel properties (jayedadaad) ka hai !!

Point-01: If $\log_b a = C \Leftrightarrow a = b^C$

Point-02: $\log_a m + \log_a n = \log_a(mn)$
 $\log_a m + \log_a n + \log_a p = \log_a(mnp)$

Point-03: $\log_a m - \log_a n = \log_a\left(\frac{m}{n}\right)$

Point-04: (i) $\log_{(a)^q}(m)^p = \frac{p}{q} \log_a m$

(ii) $\log_a(m)^{2n} = 2n \log_a |m|$

Point-05: Base change property:

(i) Apni marzi ka base: $\log_b a = \frac{\log_c a}{\log_c b}$

(ii) Jab log ki terms multiplication mein ho

$\log_b a \cdot \log_c b = \log_c a$

(iii) Log wali term ka reciprocal: $\log_b a = \frac{1}{\log_a b}$

Point-06: $(a)^{\log_c b} = (b)^{\log_c a}$

Also: (i) $(a)^{k \log_c b} = b^{k \log_c a}$

(ii) $(v_1)^{v_2} = (e)^{v_2 \ln(v_1)}$

Memorize Following: $\log_{10} x = \frac{\ln x}{2.303}$

$\ln 2 = 0.693,$

$\ln 5 = 1.609,$

$\ln 3 = 1.098,$

$\ln 7 = 1.945,$

Note

Line of thinking

- (i) Try to make "base" same every where.
- (ii) Terms ko halka krne ki koshish kro.
- (iii) Property khud bta degi ki kaunsi lagegi.

Q. Simplify following:

(i) $\log_{1/3} \sqrt[4]{729 \cdot \sqrt[3]{9^{-1} \cdot 27^{-4/3}}}$

(ii) $\frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5}$ ★★ ★

Sol. (i) $\log_{1/3} \sqrt[4]{729 \cdot \sqrt[3]{9^{-1} \cdot 27^{-4/3}}}$
 $= \log_{1/3} \left(\sqrt[4]{729 \times \frac{1}{9}} \right) = \log_{1/3} (3^4)^{\frac{1}{4}} = \log_{1/3} 3$
 $= -1 \text{ fnf}$

(ii) $\log_5 250 = \log_5 125 + \log_5 2 = 3 + \log_5 2$
 $\log_5 10 = \log_5 5 + \log_5 2 = 1 + \log_5 2$

$\log_{1250} 5 = \frac{1}{\log_5 1250} = \frac{1}{\log_5 625 + \log_5 2}$

$= \frac{1}{4 + \log_5 2}$

$\log_{50} 5 = \frac{1}{\log_5 50} = \frac{1}{\log_5 25 + \log_5 2}$

$= \frac{1}{2 + \log_5 2}$

मान लो: $\log_5 2 = \alpha$

$\Rightarrow \frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5} = \frac{(3 + \alpha)}{\left(\frac{1}{2 + \alpha}\right)} - \frac{(1 + \alpha)}{\left(\frac{1}{4 + \alpha}\right)}$

$= (3 + \alpha)(2 + \alpha) - (1 + \alpha)(4 + \alpha)$
 $= 6 + 2\alpha + 3\alpha + \alpha^2 - 4 - 4\alpha - \alpha - \alpha^2$
 $= 2 \text{ fnf}$

Acidic Question

Q. Value of: $a^{\sqrt{\log_a b}} - b^{\sqrt{\log_b a}} =$ _____.

(A) $\sqrt{a} - \sqrt{b}$

(B) $b - a$

(C) $\sqrt{b} - \sqrt{a}$

(D) Zero

Sol. $(a)^{\sqrt{\log_a b}} - (a)^{\frac{\log_a b}{\sqrt{\log_a b}}} = (a^{\log_a b})^{\frac{1}{\sqrt{\log_a b}}}$

$= (b^{\log_b a})^{\frac{1}{\sqrt{\log_a b}}} = (b)^{\sqrt{\log_b a}}$

$\Rightarrow (a)^{\sqrt{\log_a b}} - (b)^{\sqrt{\log_b a}} = 0 \text{ fnf}$

CHALLENGER QUESTION

Q. Simplify following:

$\sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16}$
 $- \log_2 12 \cdot \log_2 48 =$ _____.

Sol. $\sqrt{\log_2 3 \cdot (2 + \log_2 3)(4 + \log_2 3)(6 + \log_2 3) + 16}$
 $- (2 + \log_2 3)(4 + \log_2 3)$

Put: $\log_2 3 = t$

$\Rightarrow \sqrt{t(2+t)(4+t)(6+t) + 16} - (2+t)(4+t)$

$$\Rightarrow \sqrt{(t^2 + 6t)(t^2 + 6t + 8) + 16} - (t^2 + 6t + 8)$$

$$m = t^2 + 6t$$

$$\Rightarrow \sqrt{m(m+8) + 16} - (m+8)$$

$$= \sqrt{(m+4)^2} - (m+8) = |m+4| - (m+8)$$

$$\text{as } (m > 0) \\ (m+4) - (m+8) = -4 \text{ fnf.}$$

Question [JEE (Adv.)-2018]

Q. The value of $((\log_2 9)^2)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is _____.

$$\text{Sol. } (7^{\frac{1}{2}})^{\log_7 4} = 4^{\frac{1}{2} \log_7 7} = 4^{\frac{1}{2}} = 2$$

$$\text{Now, } ((\log_2 9)^2)^{\frac{1}{\log_2(\log_2 9)}}$$

$$\text{put: } \log_2 9 = t$$

$$(t^2)^{\frac{1}{\log_2 t}} = (t^2)^{\log_t 2} = 2^{2 \log_t t} = 2^2 = 4$$

$$\text{fnf} = 4 \times 2 = 8$$

LOGARITHMIC EQUATIONS

Check your solution by re-substitution in original equation

Variable agar power mein ho to $\Rightarrow$ Take "log both side"

Any expression/term is repeating $\Rightarrow$ assume it as 't'.

Note

$$\text{If } \{a + b = cd\}$$

Take $\log_{10}()$ both sides:

$$\Rightarrow \log a + \log b = \log c \log d \rightarrow \text{Sushmita: (X)}$$

$$\Rightarrow \log(a+b) = \log(cd) \rightarrow \text{Sunita: (V)}$$

Question [JEE Main-2021]

Q. The number of solutions of the equation $\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x+1)^2 - 4 = 0, x > 0$, is _____.

$$\text{Sol. } 2x^2 + 7x + 5 = (x+1)(2x+5)$$

$$\text{Given eqn: } \log_{\alpha} \beta + \log_{\beta} (\alpha)^2 - 4 = 0$$

$$\Rightarrow \log_{\alpha} \alpha + \log_{\alpha} \beta + 2 \log_{\beta} \alpha - 4 = 0$$

$$\Rightarrow \log_{\alpha} \beta + 2 \log_{\beta} \alpha - 3 = 0$$

$$\text{put: } \log_{\beta} \alpha = t \Rightarrow \log_{\alpha} \beta = \frac{1}{t}$$

$$\frac{1}{t} + 2t - 3 = 0 \Rightarrow 2t^2 - 3t + 1 = 0$$

$$\Rightarrow (2t-1)(t-1) = 0 \Rightarrow t = 1/2, \text{ or } 1$$

$$\Rightarrow \log_{\beta} \alpha = \frac{1}{2} \text{ or } 1 \Rightarrow \alpha = \sqrt{\beta} \text{ or } \alpha = \beta$$

$$\text{for } \alpha = \sqrt{\beta} \Rightarrow (x+1) = \sqrt{2x+5} \Rightarrow \text{SBS: } x = \pm 2$$

$$\text{for } \alpha = \beta \Rightarrow x+1 = 2x+5 \Rightarrow x = -4$$

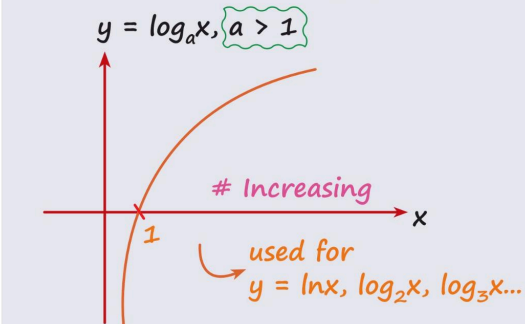
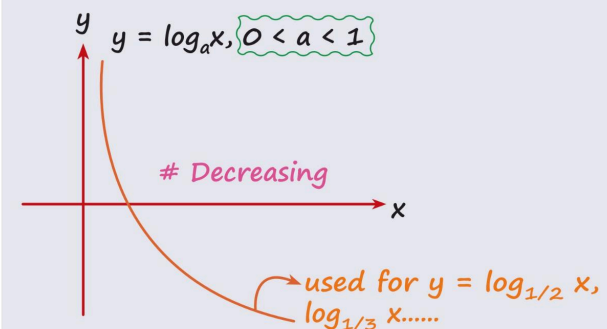
$\Rightarrow$ Only 2 is satisfying, hence fnf = 1

DOMAIN, RANGE & GRAPH OF LOG

$$y = f(x) = \log_a x, a > 0, a \neq 1$$

$$x \in (0, \infty) \rightarrow \text{Domain}$$

$$y \in (-\infty, \infty) \rightarrow \text{Range}$$



MODULE CHALLENGER QUESTION

Q. Find the number of integral solution of the equation

$$\log_{\sqrt{x}}(x + |x - 2|) = \log_x(5x - 6 + 5|x - 2|)$$

Sol. $\log_x(x + |x - 2|)^2 = \log_x(5x - 6 + 5|x - 2|)$
 $(x + |x - 2|)^2 = 5(x + |x - 2|) - 6$

put: $x + |x - 2| = m$

$$m^2 = 5m - 6$$

$$\Rightarrow m^2 - 5m + 6 = 0 \Rightarrow (m - 2)(m - 3) = 0$$

$$\Rightarrow m = 2, 3$$

Case-I: $x + |x - 2| = 2$

$x \geq 2$ $|x - 2| = 2 - x$ $x - 2 = 2 - x$ $2x = 4$ $x = 2$
 $x < 2$ all are solⁿ.
 $-(x - 2) = 2 - x$

Case-II:

$x \geq 2 \Rightarrow x - 2 = 3 - x$ For $x < 2 \Rightarrow \phi$
 $\Rightarrow x = \frac{5}{2}$ (✓)

from C-I U C-II

$\Rightarrow x = 2$ is only integer satisfying.

LOG INEQUALITY

Point-01: If $\text{base} > 1 \Rightarrow \log$ hatega या lagega to sign of inequality will remain same.

Point-02: If $0 < \text{base} < 1 \Rightarrow \log$ hatega या to sign of inequality will reversed.

Point-03: Base is variable $\Rightarrow$ Make cases according to point - 01 & 02.

Point-04: Before final answer take intersection with Domain of Log.

Blue बादल (KKPNCC)

Log - wali Inequality

defining part $\uparrow$ Solution Part

Question

Q. Solve: $\log_{1/3}(x^2 - 7) \leq -2$

Sol. $\log_{1/3}(x^2 - 7) \leq -2$

$$\Rightarrow x^2 - 7 \geq 9$$

$$\Rightarrow x^2 - 16 \geq 0$$

$$\Rightarrow (x - 4)(x + 4) \geq 0$$



fnf: solⁿ. $\cap$ Def:



$$\Rightarrow x \in (-\infty, -4] \cup [4, \infty) \text{ fnf.}$$

When base is variable

Consider two cases:

Case-I: Base > 1

Case-II: $0 < \text{Base} < 1$

Solution: (C-I) $\cup$ (C-II)

Question

Q. Find the solution set of inequality,

$$\log_{x+3}(x^2 - x) < 1$$

Sol. $\log_{(x+3)}(x^2 - x) < 1$

C-I: base > 1

$$\Rightarrow x + 3 > 1$$

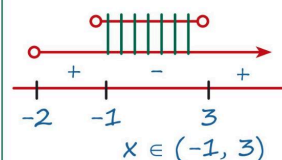
$$\Rightarrow x > -2$$

$$\log_{(x+3)}(x^2 - x) < 1$$

$$\Rightarrow x^2 - x < (x + 3)$$

$$\Rightarrow x^2 - 2x - 3 < 0$$

$$\Rightarrow (x - 3)(x + 1) < 0$$



C-II: $0 < \text{base} < 1$

$$\Rightarrow 0 < x + 3 < 1$$

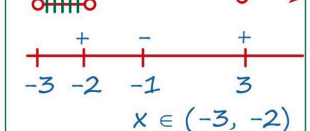
$$\Rightarrow -3 < x < -2$$

$$\log_{(x+3)}(x^2 - x) < 1$$

$$\Rightarrow x^2 - x > (x + 3)$$

$$\Rightarrow x^2 - 2x - 3 > 0$$

$$\Rightarrow (x - 3)(x + 1) > 0$$

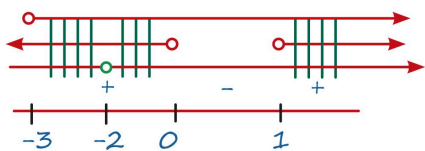


$$\Rightarrow \text{Solution (C-I U C-II)} \Rightarrow x \in (-3, -2) \cup (-1, 3)$$

Defining:

$$x^2 - x > 0 \cap x + 3 > 0 \cap x + 3 \neq 1$$

$$\Rightarrow x(x - 1) > 0 \cap x > -3 \cap x \neq -2$$



$$x \in (-3, -2) \cup (-2, 0) \cup (1, \infty)$$

fnf: Solution $\cap$ Defining

$$x \in (-3, -2) \cup (-1, 0) \cup (1, 3) \text{ fnf.}$$

Question [JEE Main-2023] ★★★★★

Q. The number of integral solution x of

$$\log_{x+\frac{7}{2}} \left(\frac{x-7}{2x-3} \right)^2 \geq 0 \text{ is}$$

(A) 6

(B) 8

(C) 5

(D) 7

Sol. Case-I: $x + \frac{7}{2} > 1 \Rightarrow x > \frac{-5}{2}$

$$\Rightarrow \log_{\left(x+\frac{7}{2}\right)} \left(\frac{x-7}{2x-3} \right)^2 \geq 0$$

$$\Rightarrow \left(\frac{x-7}{2x-3} \right)^2 \geq 1$$

$$\Rightarrow \frac{(x-7)^2}{(2x-3)^2} - 1 \geq 0$$

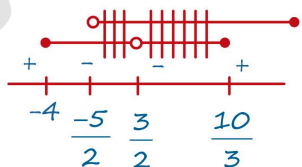
$$\Rightarrow \frac{(3x-10)(x+4)}{(2x-3)^2} \leq 0$$

$$x \in \left(\frac{-5}{2}, \frac{3}{2} \right) \cup \left(\frac{3}{2}, \frac{10}{3} \right]$$

Case-II: $0 < x + \frac{7}{2} < 1 \Rightarrow \left(\frac{x-7}{2x-3} \right)^2 \leq 1 \Rightarrow \phi$

Defining:

$$x + \frac{7}{2} > 0, 0 + \frac{7}{2} \neq 1 \cap \left(\frac{x-7}{2x-3} \right)^2 > 0$$



Defining solution: $x \in \left(\frac{-5}{2}, \frac{3}{2} \right) \cup \left(\frac{3}{2}, \frac{10}{3} \right]$

No. of intergeral solution = 6 fnf.

EXPONENTIAL FUNCTION

Domain, range & graph:

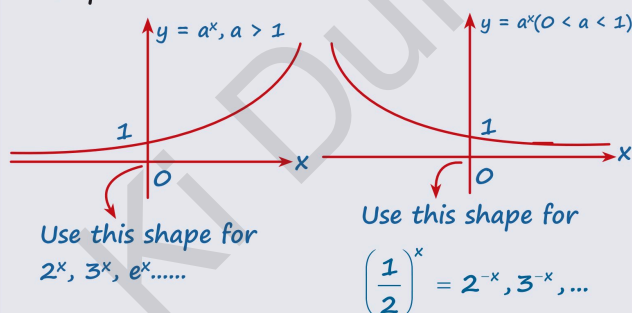
$$y = a^x \quad (a > 0, a \neq 1)$$

$$2^x, 3^x, 4^x, e^x, \dots$$

Domain $(x) \in \mathbb{R}$

Range $(y) \in \mathbb{R}^+$ or $(0, \infty)$

Graph:



MOST IMPORTANT NOTEL

$(a)^b \rightarrow$ power (b) बदल रहा hai या variable है by forming an interval $b \in (p, q)$, then $a > 0$.

LAW OF INDICES (POWERS)

1. $a^0 = 1, (a \neq 0)$

2. $a^{-m} = \frac{1}{a^m}, (a \neq 0)$

3. $a^m \cdot a^n = a^{m+n}$

4. $\frac{a^m}{a^n} = a^{m-n}$

5. $(a^m)^n = a^{mn} = (a^n)^m$

6. $\sqrt[p]{\sqrt[q]{a}} = a^{\frac{1}{pq}} = \sqrt[q]{\sqrt[p]{a}}$

7. $a^{\frac{1}{p}} = \sqrt[p]{a}$

8. $a^m b^m = (ab)^m$

9. $\left(\frac{a}{b} \right)^m = \frac{a^m}{b^m}$

10. $a^{b^c} = (a^{b^c})$

Question

Q. If $a^x = \sqrt{b}, b^y = \sqrt[3]{c}$ and $c^z = \sqrt{a}$, then the value of xyz is

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{6}$

(D) $\frac{1}{12}$

Sol. $a^x = b^{\frac{1}{2}}$, $b^y = c^{\frac{1}{3}}$, $c^z = a^{\frac{1}{2}}$

$\downarrow$ $\downarrow$ $\downarrow$

$c^{2zx} = c^{\frac{1}{6y}}$ $b = c^{\frac{1}{3y}}$ $c^{2z} = a$

$2zx = \frac{1}{6y}$ $\frac{1}{b^2} = \left(c^{\frac{1}{3y}}\right)^{\frac{1}{2}}$ $c^{2zx} = a^x$

fnf: $xyz = \frac{1}{12}$

EXPONENTIAL EQUATION

1. $a^x = a^y \Rightarrow x = y$
2. $a^x = b^y$ (Take log both side & proceed)

Question

Q. Solve: $4^{\sqrt{x^2-2}+x} - 5 \cdot 2^{x-1+\sqrt{x^2-2}} = 6$

Sol. $4^{\sqrt{x^2-2}+x} - 5 \cdot 2^{x-1+\sqrt{x^2-2}} = 6$

$4^{\sqrt{x^2-2}+x} - 5 \cdot \frac{2^{x+\sqrt{x^2-2}}}{2} = 6$

मान लो: $2^{x+\sqrt{x^2-2}} = t$

$t^2 - \frac{5}{2}t - 6 = 0$

$2t^2 - 5t - 12 = 0 \Rightarrow (2t+3)(t-4) = 0$

$\rightarrow$ rejected

$t = -\frac{3}{2}, 4 \Rightarrow 2^{x+\sqrt{x^2-2}} = 2^2$

$x + \sqrt{x^2-2} = 2 \Rightarrow \sqrt{x^2-2} = 2-x$

$\Rightarrow x = \frac{3}{2}$ fnf

EXPONENTIAL INEQUALITY

If $a^x \geq a^y \Rightarrow x \geq y, a > 1$
 $\Rightarrow x \leq y, 0 < a < 1$

Aap dono traf log apply krke bhi proceed kr sktte ho.

Question

A. Solve: $\left(\frac{3}{4}\right)^{6x+10-x^2} < \frac{27}{64}$

B. Solve: $2(25)^x - 5(10^x) + 2(4^x) \geq 0$ ★★★★★

Sol. (A) $\left(\frac{3}{4}\right)^{6x+10-x^2} < \left(\frac{3}{4}\right)^3$

$\Rightarrow 6x+10-x^2 > 3$

$\Rightarrow 0 > x^2 - 6x - 7$

$\Rightarrow 0 > (x-7)(x+1)$

$\begin{array}{c} + \quad - \quad + \\ -1 \quad 7 \end{array}$ fnf.

(B) $2(25)^x - 5(10^x) + 2(4^x) \geq 0$

$\Rightarrow 2(5)^{2x} - 5 \cdot 5^x \cdot 2^x + 2 \cdot 2^{2x} \geq 0$

put: $5^x = m, 2^x = n$

$2m^2 - 5mn + 2n^2 \geq 0$

$\Rightarrow 2m^2 - 4mn - mn + 2n^2 \geq 0$

$\Rightarrow (m-2n)(2m-n) \geq 0$

$(5^x - 2 \cdot 2^x)(2 \cdot 5^x - 2^x) \geq 0$

$\begin{array}{c} \oplus \quad \ominus \quad \oplus \\ -\alpha \quad \alpha \end{array}$

$x \in (-\infty, -\alpha] \cup [\alpha, \infty)$ where $\alpha = \frac{\log_5 2}{1 - \log_5 2}$

Question [JEE Main-2019, IIT-JEE-1997]

Q. Find the set of all solutions of the equation

$2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1.$

Sol. $2^{|y|} = |2^{y-1} + 1| + |2^{y-1} - 1|$

$\Rightarrow$ C-I: $y \geq 0$

$2^{|y|} = |2^{y-1} - 1| + |2^{y-1} + 1|$

$|\alpha + \beta| = |\alpha| + |\beta|$

Solution is $\alpha\beta \geq 0$

$(2^{y-1} + 1)(2^{y-1} - 1) \geq 0$

$2^{y-1} \geq 1 \Rightarrow y-1 \geq 0 \Rightarrow y \geq 1$

C-I: $y \geq 0 \cap y \geq 1 \Rightarrow y \geq 1$

C-II: $y < 0$

$2^{-y} = |2^{y-1} + 1| + |2^{y-1} - 1|$

$2^{-y} = 2^{y-1} + 1 + 1 - 2^{y-1}$

$2^{-y} = 2^1$

$-y = 1 \Rightarrow y = -1$

C-I $\cup$ C-II $\Rightarrow y \in [1, \infty) \cup \{-1\}$ fnf

JEE wale पछते hai kyunki ismein cases सोचने pdte hai

Critical Thinking Question [JEE Main-2016]

$$(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1 \text{ is}$$

- Case-II: $v_2 = 0$
- $$x^2 + 4x - 60 = 1$$
- $$(x - 6)(x + 10) = 0$$
- $x = -10, 6$
- Check: $x = -10$
- $$\text{LHS} = (155)^0 = 1$$
- $$x = 6$$
- $$\text{LHS} = (11)^0 = 1$$

$$x^2 - 5x + 5 = -1 \Rightarrow x^2 - 5x + 6 = 0$$
$$(x - 2)(x - 3) = 0 \Rightarrow x = 2, 3$$

Check: $x = 2 \Rightarrow \text{LHS} = (-1)^{-48} = \frac{1}{(-1)^{48}} = 1$ ✓

$$x=3 \Rightarrow LHS = (-1)^{-3q} = \frac{1}{(-1)^{3q}} = \frac{1}{-1} = -1 \quad (\text{X})$$

$\Rightarrow x = 1, 4, -10, 6, 2$
 $\Rightarrow$ Sum of all values = 3 fnf.

Note-01

If $a : b = c : d \Rightarrow \frac{a}{b} = \frac{c}{d}$

If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}, \dots$ then

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = \frac{a + c + e + \dots}{b + d + f + \dots}$$

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}, \dots$ then

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = \frac{xa + yc + ze + \dots}{xb + yd + zf + \dots}$$

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}, \dots$

$$\text{then each} = \left(\frac{xa^n + yc^n + ze^n}{xb^n + yd^n + zf^n} \right)^{1/n}$$

Q. Let a , b and c be distinct non zero real number such that $\frac{1-a^3}{a} = \frac{1-b^3}{b} = \frac{1-c^3}{c}$.

Sol. मान लो: $\frac{1-a^3}{a} = \frac{1-b^3}{b} = \frac{1-c^3}{c} = k$

$$\frac{1-a^3}{a} = k \Rightarrow 1-a^3 = ak$$

Diagram illustrating the combination of three cubic equations:

$$\begin{cases} a^3 + ak - 1 = 0 \\ b^3 + bk - 1 = 0 \\ c^3 + ck - 1 = 0 \end{cases} \rightarrow \text{Combine} \rightarrow \begin{cases} \text{Cubic} \\ x^3 + kx - 1 = 0 \\ x^3 + 0x^2 + kx - 1 \leq 0 \\ \begin{cases} a + b + c = 0 \\ abc = 1 \end{cases} \end{cases}$$

The diagram shows three equations in curly braces on the left, an arrow pointing to the word "Combine", and then a large curly brace containing the resulting cubic equation, its expanded form, and the symmetric sums of the roots a, b, c . Arrows also point from the roots a, b, c to the symmetric sum equations.

$$\Rightarrow 10(a^3 + b^3 + c^3) = 10(3abc) = 30 \text{ fnf}$$

3

Trigonometric Ratios & Identities

BASIC OF TRIGONOMETRY

Point-01: Angles

Measurement of Angles

⇒ Degree system & Radian (circular system)

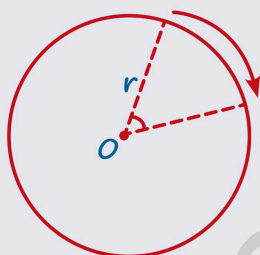
⇒ π radian = 180°

⇒ 1 radian = $\frac{180^\circ}{\pi}$

1 radian = $\frac{180^\circ}{\pi} = 57^\circ 16'$ approximately

⇒ $1^\circ = \left(\frac{\pi}{180}\right)$ radian

$1^\circ = \frac{\pi}{180}$ radian = 0.01746 radian approximately



(RAR) (खून में बसा लो)

$30^\circ = \frac{\pi}{6}$	$45^\circ = \frac{\pi}{4}$	$60^\circ = \frac{\pi}{3}$
$90^\circ = \frac{\pi}{2}$	$120^\circ = \frac{2\pi}{3}$	$135^\circ = \frac{3\pi}{4}$
$150^\circ = \frac{5\pi}{6}$	$180^\circ = \pi$	$210^\circ = \frac{7\pi}{6}$
$225^\circ = \frac{5\pi}{4}$	$240^\circ = \frac{4\pi}{3}$	$270^\circ = \frac{3\pi}{2}$
$300^\circ = \frac{5\pi}{3}$	$315^\circ = \frac{7\pi}{4}$	$330^\circ = \frac{11\pi}{6}$

Note

(1) $90^\circ, 180^\circ, 270^\circ \dots \Rightarrow$ Allied angles

(2) If $\alpha + \beta = \pi \Rightarrow \alpha$ & β Supplementary angles

(3) If $\alpha + \beta = \frac{\pi}{2} \Rightarrow \alpha$ & β complementary angles

RAR

$\pi \approx 3.14, \frac{\pi}{2} \approx 1.57, \frac{\pi}{4} \approx 0.78$

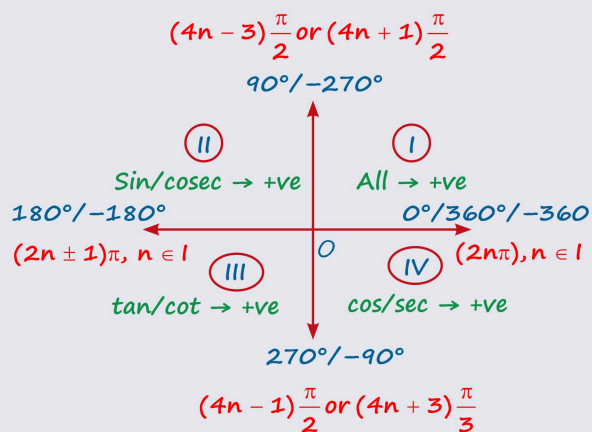
Note

Angle measured in anticlockwise direcⁿ. $\equiv (+ve)$



Angle measured in clockwise direcⁿ. $\equiv (-ve)$

Point-02: Quadrant



Point-03: Purana Pyaar (Class X wala)

Sine	Cosine	Tangent
$\frac{P}{H}$	$\frac{B}{H}$	$\frac{P}{B}$
Cosecant	Secant	Cotangent

TRIGONOMETRIC IDENTITIES

- (1) $(\sin^2\theta + \cos^2\theta) = 1$
 (2) $(\sec^2\theta - \tan^2\theta) = 1$
 $\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$
 (3) $(\operatorname{cosec}^2\theta - \cot^2\theta) = 1$
 $\Rightarrow (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$

Question

Q. If $\theta \in I$ quadrant & $\sec\theta + \tan\theta = \frac{3}{2}$. Then value of $\cos\theta$ is _____.

Sol. Given: $\sec\theta + \tan\theta = \frac{3}{2}$... (i)
 $\Rightarrow \sec\theta - \tan\theta = \frac{2}{3}$... (ii)

add _____

$$2\sec\theta = \frac{13}{6} \Rightarrow \sec\theta = \frac{13}{12}$$

$$\cos\theta = \frac{12}{13} \text{ fnf.}$$

Question [JEE Main-2019]

Q. Let $f_k = \frac{1}{k} (\sin^k x + \cos^k x)$ for $k = 1, 2, 3, \dots$

then for all $x \in \mathbb{R}$, the value of $f_4(x) - f_6(x)$ is equal to

- (A) $\frac{1}{12}$ (B) $\frac{1}{4}$
 (C) $-\frac{1}{12}$ (D) $\frac{5}{12}$

Sol. $\Rightarrow \sin^2 x + \cos^2 x = 1$

SBS: $\sin^4 x + \cos^4 x + 2\sin^2 x \cos^2 x = 1$... (i)

$$\Rightarrow \sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3$$

$$= (\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x)$$

$$\sin^6 x + \cos^6 x = 1 - 3\sin^2 x \cos^2 x \quad \dots (ii)$$

$$\text{now; } f_4(x) = \frac{1}{4} (1 - 2\sin^2 x \cos^2 x)$$

$$\text{and, } f_6(x) = \frac{1}{6} (1 - 3\sin^2 x \cos^2 x)$$

$$\Rightarrow f_4(x) - f_6(x) = \frac{1}{4} - \frac{1}{6} = \frac{1}{12} \text{ fnf.}$$

Acidic Question

Q. The value of

$\log_{(\sin^2 x + \cos^4 x + 2)} (\cos^2 x + \sin^4 x + 2)$ is

- (A) 1 (B) -1 (C) 0 (D) 2

Sol. $\cos^2 x + \sin^4 x + 2 = \cos^2 x + (1 - \cos^2 x)^2 + 2$
 $= \cos^2 x + 1 + \cos^4 x - 2\cos^2 x + 2$
 $= 1 - \cos^2 x + \cos^4 x + 2$
 $= \sin^2 x + \cos^4 x + 2$
 $\Rightarrow \text{Req. ans} = 1 \text{ fnf.}$

Note

Dealing with -ve sign

$\sin(-\theta) = -\sin\theta$	$\cot(-\theta) = -\cot(\theta)$
$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$	$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$	$\sec(-\theta) = \sec\theta$

Note

"How to reduce angle"

(1) Angle ka quadrant pta kro (θ acute imagine krte hue)

(2) Angle = $a.(90^\circ) \pm \theta$

$a \equiv \text{even} \rightarrow \text{function nahi badlega}$
 $a \equiv \text{odd} \rightarrow \text{function will change}$

$$\begin{array}{l} \sin \leftrightarrow \cos \\ \tan \leftrightarrow \cot \\ \operatorname{cosec} \leftrightarrow \sec \end{array}$$

REDUCTION FORMULA

$\sin\left(\frac{\pi}{2} - \theta\right) = +\cos\theta$	$\sin\left(\frac{\pi}{2} + \theta\right) = +\cos\theta$
$\cos\left(\frac{\pi}{2} - \theta\right) = +\sin\theta$	$\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$
$\tan\left(\frac{\pi}{2} - \theta\right) = +\cot\theta$	$\tan\left(\frac{\pi}{2} + \theta\right) = -\cot\theta$
$\cot\left(\frac{\pi}{2} - \theta\right) = +\tan\theta$	$\cot\left(\frac{\pi}{2} + \theta\right) = -\tan\theta$
$\operatorname{cosec}\left(\frac{\pi}{2} - \theta\right) = +\sec\theta$	$\operatorname{cosec}\left(\frac{\pi}{2} + \theta\right) = +\sec\theta$

$\sec\left(\frac{\pi}{2} - \theta\right) = +\operatorname{cosec} \theta$	$\sec\left(\frac{\pi}{2} + \theta\right) = -\operatorname{cosec} \theta$
$\sin(\pi - \theta) = +\sin \theta$	$\sin(\pi + \theta) = -\sin \theta$
$\cos(\pi - \theta) = -\cos \theta$	$\cos(\pi + \theta) = -\cos \theta$
$\tan(\pi - \theta) = -\tan \theta$	$\tan(\pi + \theta) = \tan \theta$
$\cot(\pi - \theta) = -\cot \theta$	$\cot(\pi + \theta) = +\cot \theta$
$\operatorname{cosec}(\pi - \theta) = +\operatorname{cosec} \theta$	$\operatorname{cosec}(\pi + \theta) = -\operatorname{cosec} \theta$
$\sec(\pi - \theta) = -\sec \theta$	$\sec(\pi + \theta) = -\sec \theta$

Examples

- (1) $\sin(2\pi - \theta) = \sin\left(\frac{4\pi}{2} - \theta\right) = -\sin \theta$
- (2) $\tan\left(\frac{3\pi}{2} + \theta\right) = -\cot \theta$
- (3) $\operatorname{cosec}(5\pi - \theta) = \operatorname{cosec}\left(\frac{10\pi}{2} - \theta\right) = +\operatorname{cosec} \theta$
- (4) $\sec(2\pi + \theta) = \sec \theta$
- (5) $\cos\left(\frac{5\pi}{2} - \theta\right) = +\sin \theta$
- (6) $\sin\left(\theta - \frac{3\pi}{2}\right) = -\sin\left(\frac{3\pi}{2} - \theta\right) = -(-\cos \theta) = +\cos \theta$
- (7) $\cos\left(\theta - \frac{\pi}{2}\right) = \cos\left(-\left(\frac{\pi}{2} - \theta\right)\right) = \cos\left(\frac{\pi}{2} - \theta\right) = +\sin \theta$

VALUE OF TRIGONOMETRIC FUNCTION

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined

	0°	30°	45°	60°	90°
cot	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
cosec	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Q. Find value of following:

$$(1) \cos(240^\circ) = \cos(2 \times 90^\circ + 60^\circ) \\ = -(\cos 60^\circ) = \frac{-1}{2}$$

$$(2) \tan(-300^\circ) = -\tan(300^\circ) \\ = -\tan(3 \times 90^\circ + 30^\circ) \\ = -(-\cot 30^\circ) = +\sqrt{3}$$

$$(3) \sin\left(-\frac{11\pi}{3}\right) = -\sin \frac{11\pi}{3} = -\sin\left(4\pi - \frac{\pi}{3}\right) \\ = -\left(-\sin \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$(4) \tan\left(\frac{19\pi}{3}\right) = \tan\left(6\pi + \frac{\pi}{3}\right) = +\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$(5) \cot(-1470^\circ) = -\cot(1470^\circ) \\ = -\cot(16 \times 90^\circ + 30^\circ) \\ = -(+\cot 30^\circ) = -\sqrt{3}$$

Question

Q. Find value of following:

$$(i) \alpha = \tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ = 1$$

$$(ii) \alpha = \cos 15^\circ \cos 16^\circ \cos 17^\circ \dots \cos 175^\circ = 0$$

$$(iii) \gamma = \sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ \dots \sin^2 90^\circ = \frac{19}{2}$$

$$(iv) \delta = \cos^2 \frac{\pi}{16} + \cos^2 \frac{3\pi}{16} + \cos^2 \frac{5\pi}{16} + \cos^2 \frac{7\pi}{16} = 2$$

Question

Q. If $\operatorname{cosec} \theta - \sin \theta = m$ and $\sec \theta - \cos \theta = n$, then $(m^2 n)^{2/3} + (mn^2)^{2/3} =$

Sol. $\sec\theta - \cos\theta = n \Rightarrow \operatorname{cosec}\theta - \sin\theta = m$

$$\Rightarrow \frac{1}{\cos\theta} - \cos\theta = n$$

$$\Rightarrow \frac{\sin^2\theta}{\cos\theta} = n$$

$$\Rightarrow \sin^2\theta = n\cos\theta$$

$$\Rightarrow \sin^2\theta = n\sqrt{m\sin\theta}$$

$$\Rightarrow \sin\theta = (n\sqrt{m})^{2/3}$$

$\Rightarrow$ Square and add:

$$(\sqrt{m}n)^{4/3} + (\sqrt{n}m)^{4/3} = 1$$

$$(mn^2)^{2/3} + (nm^2)^{2/3} = 1 \text{ fnf.}$$

$$\Rightarrow \frac{1}{\sin\theta} - \sin\theta = m$$

$$\Rightarrow \frac{\cos^2\theta}{\sin\theta} = m$$

$$\Rightarrow \cos^2\theta = m\sin\theta$$

$$\Rightarrow \cos\theta = (m\sin\theta)^{\frac{1}{2}}$$

$$\Rightarrow \cos\theta = (m\sqrt{n})^{2/3}$$

COMPOUND ANGLE FORMULA

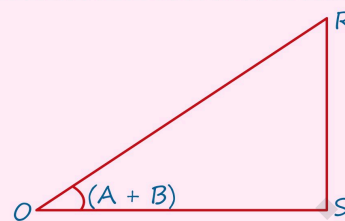
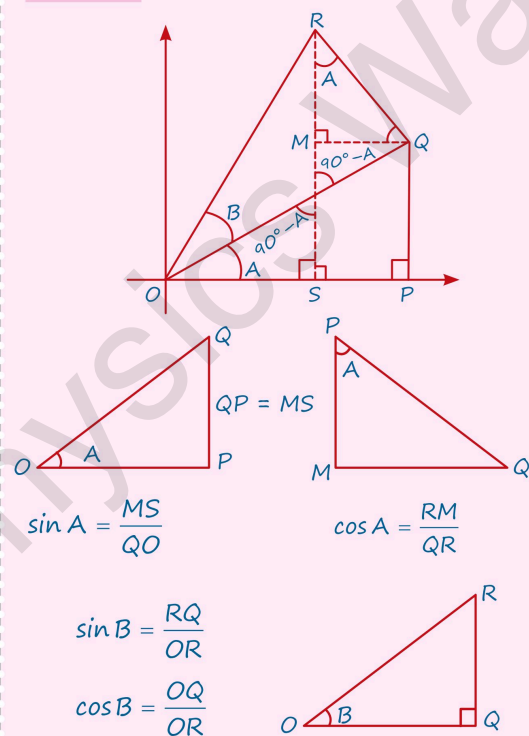
$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

Proof:



$$\Rightarrow \sin(A + B) = \frac{RS}{OR} = \frac{RM + MS}{OR}$$

$$= \frac{RM}{OR} + \frac{MS}{OR}$$

$$= \frac{RM}{QR} \cdot \frac{QR}{OR} + \frac{MS}{OQ} \cdot \frac{OQ}{OR}$$

$$\Rightarrow \sin(A + B) = \cos A \sin B + \sin A \cos B$$

HPPP

Note

Dealing with (-ve) sign & Reduction formula can be easily proved using compound angle formula.

If $A = \frac{\pi}{2}$ & $B = \theta$:

$$\cos\left(\frac{\pi}{2} + \theta\right) = \cos\frac{\pi}{2} \cos\theta - \sin\frac{\pi}{2} \sin\theta = -\sin\theta$$

$$\sin\left(\frac{\pi}{2} + \theta\right) = \sin\frac{\pi}{2} \cos\theta + \cos\frac{\pi}{2} \sin\theta = +\cos\theta$$

If $A = 0$ & $B = \theta$:

$$\sin(0 - \theta) = \sin 0 \cos\theta - \cos 0 \sin\theta = -\sin\theta$$

$$\cos(0 - \theta) = \cos 0 \cos\theta + \sin 0 \sin\theta = +\cos\theta$$

FORMULA'S FOR TANGENT & COTANGENT

$$1. \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$2. \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$3. \cot(A + B) = \left(\frac{\cot A \cot B - 1}{\cot A + \cot B} \right)$$

$$4. \cot(A - B) = \left(\frac{\cot A \cot B + 1}{\cot B - \cot A} \right)$$

Note

$$\Rightarrow \tan\left(\frac{\pi}{4} + \theta\right) = \left(\frac{1 + \tan \theta}{1 - \tan \theta}\right)$$

$$\Rightarrow \tan\left(\frac{\pi}{4} - \theta\right) = \left(\frac{1 - \tan \theta}{1 + \tan \theta}\right)$$

IMPORTANT VALUES (RAR)

$$1. \sin(15^\circ) = \sin \frac{\pi}{12} = \frac{\sqrt{3} - 1}{2\sqrt{2}} = \cos(75^\circ) = \cos \frac{5\pi}{12}$$

$$2. \sin(75^\circ) = \sin \frac{5\pi}{12} = \frac{\sqrt{3} + 1}{2\sqrt{2}} = \cos(15^\circ) = \cos \frac{\pi}{12}$$

$$3. \tan(15^\circ) = \tan \frac{\pi}{12} = 2 - \sqrt{3} = \cot(75^\circ) = \cot \frac{5\pi}{12}$$

$$4. \tan(75^\circ) = \tan \frac{5\pi}{12} = 2 + \sqrt{3} = \cot(15^\circ) = \cot \frac{\pi}{12}$$

Question [JEE में Repeating Variety] ★★

Q. If $\sin(A - B) = \frac{1}{\sqrt{10}}$, $\cos(A + B) = \frac{2}{\sqrt{29}}$,
find the value of $\tan 2A$ where A and B lie
between 0 and $\frac{\pi}{4}$.

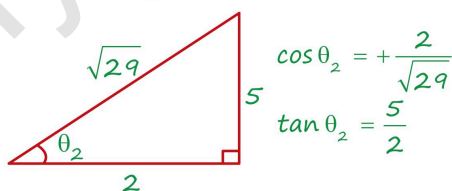
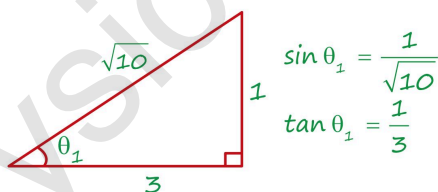
Sol.

$$0 < A < \frac{\pi}{4} \quad 0 < A < \frac{\pi}{4}$$

$$0 < B < \frac{\pi}{4} \xrightarrow{\ominus} -\frac{\pi}{4} < -B < 0$$

$$0 < A + B < \frac{\pi}{2} \quad -\frac{\pi}{4} < A - B < \frac{\pi}{4}$$

$$\theta_2 \Rightarrow \text{I quad.} \quad \theta_1 \Rightarrow \text{I quad.}$$



$$\Rightarrow \tan 2A = \tan((A + B) + (A - B))$$

$$\text{where: } \theta_1 = A - B, \theta_2 = A + B$$

$$\Rightarrow \tan 2A = \left(\frac{\tan \theta_2 + \tan \theta_1}{1 - \tan \theta_2 \times \tan \theta_1}\right)$$

$$\Rightarrow \tan 2A = \frac{\frac{5}{2} + \frac{1}{3}}{1 - \frac{1}{3} \times \frac{5}{2}} = \frac{17}{1} = 17 \text{ fnf.}$$

$$\text{Similarly, } \tan(2B) = \tan((A + B) - (A - B)).$$

Question

Q. Value of: $(\tan 100^\circ + \tan 125^\circ + \tan 100^\circ \tan 125^\circ)$ is

- (A) 0 (B) -1
(C) 2 (D) 1

$$\text{Sol. } 100^\circ + 125^\circ = 225^\circ$$

$$\Rightarrow \tan(100^\circ + 125^\circ) = \tan(225^\circ)$$

$$= \tan(2 \times 30^\circ + 45^\circ)$$

$$= \tan(45^\circ)$$

$$\Rightarrow \frac{\tan 100^\circ + \tan 125^\circ}{1 - \tan 100^\circ \tan 125^\circ} = 1$$

$$\Rightarrow \tan 100^\circ + \tan 125^\circ = 1 - \tan 100^\circ \tan 125^\circ$$

$$\Rightarrow \tan 100^\circ + \tan 125^\circ + \tan 100^\circ \tan 125^\circ = 1 \text{ fnf.}$$

Black बादल (KKPNCC)

Class X वाली सोच: $\sin$ & $\cos$ में बदलो

IIT वाली सोच: $\rightarrow \tan$ & $\cot$ के case में

Angles से Shuru करो

Question Trigonometry badi Jalim hai.

Q. Evaluate each part carefully

(i) If $\cos 18^\circ - \sin 18^\circ = k \sin 27^\circ$ & $\alpha = k^2$

(ii) $\beta = \cot 16^\circ \cot 44^\circ + \cot 44^\circ \cot 76^\circ - \cot 76^\circ \cot 16^\circ$

(iii) If $\cos \alpha + \cos \beta + \cos \gamma = \sin \alpha + \sin \beta + \sin \gamma = 0$, then $\gamma = \cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha)$

Find value of $\left[\alpha + \beta + \frac{\gamma}{\beta}\right] = \underline{\hspace{2cm}}$
(where [.] GIF)

Sol. (i) $18^\circ + 27^\circ = 45^\circ$
 $27^\circ = 45^\circ - 18^\circ$
 $\Rightarrow \sin 27^\circ = \sin(45^\circ - 18^\circ)$
 $\Rightarrow \sin 27^\circ = \frac{1}{\sqrt{2}} \cdot \cos 18^\circ - \frac{1}{\sqrt{2}} \sin 18^\circ$
 $\Rightarrow \sqrt{2} \sin 27^\circ = \cos 18^\circ - \sin 18^\circ$
 $\alpha = k^2 = 2$

(ii) $\cot A \cot B = \cot(A+B)(\cot A + \cot B) + 1$
 $\cot 16^\circ \cot 44^\circ = \frac{1}{\sqrt{3}} (\cot 16^\circ + \cot 44^\circ) + 1$... (1)
 $\cot 44^\circ \cot 75^\circ = -\frac{1}{\sqrt{3}} (\cot 44^\circ + \cot 76^\circ) + 1$... (2)
 $-\cot A \cot B = -\cot(A-B)(\cot B - \cot A) + 1$
 $-\cot 76^\circ \cot 16^\circ = -\frac{1}{\sqrt{3}} (\cot 16^\circ - \cot 76^\circ) + 1$... (3)

add (1), (2) and (3) $\Rightarrow \beta = 3$

(iii) $\cos \alpha + \cos \beta + \cos \gamma = 0$
SBS: $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + 2 \cos \alpha \cos \beta + 2 \cos \beta \cos \gamma + 2 \cos \gamma \cos \alpha = 0$... (1)
 $\sin \alpha + \sin \beta + \sin \gamma = 0$
SBS: $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma + 2 \sin \alpha \sin \beta + 2 \sin \beta \sin \gamma + 2 \sin \gamma \sin \alpha = 0$... (2)

Add _____
 $\Rightarrow 1 + 1 + 1 + 2 \cos(\alpha - \beta) + 2 \cos(\beta - \gamma) + 2 \cos(\gamma - \alpha) = 0$
 $\gamma = -\frac{3}{2}$ **DIBY**
 $\Rightarrow \left[\alpha + \beta + \frac{\gamma}{\beta} \right] = \left[2 + 3 - \frac{1}{2} \right] = 4 \text{ fnf.}$

Question [JEE Main-2019] **DIBY**

Q. If $\cos(\alpha + \beta) = \frac{3}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$ and

$0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to

Ans. 63/16

Question [JEE Main-2021]

Q. If $15\sin^4 \alpha + 10\cos^4 \alpha = 6$, for some $\alpha \in \mathbb{R}$, then the value of $27\sec^6 \alpha + 8\csc^6 \alpha$
(A) 350 (B) 500 (C) 400 (D) 250

Sol. $15\sin^4 \alpha + 10\cos^4 \alpha = 6$
 $\Rightarrow 15(\sin^2 \alpha)^2 + 10(1 - \sin^2 \alpha)^2 = 6$
मान लो: $\sin^2 \alpha = t$
 $\Rightarrow 15(t)^2 + 10(1 - t)^2 = 6$
 $\Rightarrow 25t^2 - 20t + 4 = 0$
 $\Rightarrow (5t - 2)^2 = 0 \Rightarrow t = \frac{2}{5}$
 $\Rightarrow \sin^2 \alpha = \frac{2}{5} \Rightarrow \cos^2 \alpha = 1 - \frac{2}{5} = \frac{3}{5}$
 $\Rightarrow 27\sec^6 \alpha + 8\csc^6 \alpha = 27 \times \left(\frac{5}{3}\right)^3 + 8 \left(\frac{5}{2}\right)^3$
 $= \frac{27 \times 125}{27} + \frac{8 \times 125}{8} = 250 \text{ fnf.}$

IMPORTANT RESULT

Result-01: If $A + B = \frac{3}{4}$, then:

$(1 + \tan A)(1 + \tan B) = 2$

Result-02:

$\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - (\tan A \tan B + \tan A \tan C + \tan B \tan C)}$

Result-03:

$\tan(\theta_1 + \theta_2 + \theta_3 + \dots \theta_n) = \left(\frac{S_1 - S_3 + S_5 - S_7 + \dots}{1 - S_2 + S_4 - S_6 + \dots} \right)$

Q. If $A + B = 45^\circ$, then $(\cot A - 1)(\cot B - 1) = \underline{\hspace{2cm}}$.

Sol. 2 **DIBY**

TRANSFORMATION FORMULAS

Transforming Sum/Difference into product:

$\sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$

$\sin C - \sin D = 2 \sin \left(\frac{C-D}{2} \right) \cos \left(\frac{C+D}{2} \right)$

$\cos C + \cos D = 2 \cos \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$

$\cos C - \cos D = -2 \sin \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$

Transforming Product into Sum/Difference:

$$2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A \sin B = \cos(A-B) - \cos(A+B)$$

Black बाल (KKPNCC)

Trigonometry ki सबसे बड़ी Simplification

Trigonometric Terms in (+) or (-) $\xrightarrow{\text{Convert into}}$ Multiplication

Trigonometric Terms in multiplication $\xrightarrow{\text{Convert into}}$ (+) or (-)

Question

Q. Prove the following

$$\frac{\sin A - \sin 5A + \sin 9A - \sin 13A}{\cos A - \cos 5A - \cos 9A + \cos 13A} = \cot 4A$$

$$\text{Sol. } N^r = \sin A - \sin 5A + \sin 9A - \sin 13A$$

$$= (\sin A + \sin 9A) - (\sin 5A + \sin 13A)$$

$$= 2\sin 5A \cos 4A - 2\sin 9A \cos 4A$$

$$= 2\cos 4A (\sin 5A - \sin 9A)$$

$$= 2\cos 4A (2\cos 7A \sin(-2A))$$

$$D^r = (\cos A + \cos 13A) - (\cos 5A + \cos 9A)$$

$$= 2\cos 7A \cos 6A - (2\cos 7A \cos 2A)$$

$$= 2\cos 7A (\cos 6A - \cos 2A)$$

$$= 2\cos 7A (-2\sin 4A \sin 2A)$$

$$\frac{N^r}{D^r} = \frac{2\cos 4A \cdot 2\cos 7A \sin(-2A)}{2\cos 7A \cdot (-2\sin 4A \sin 2A)} = \cot 4A$$

HHPP

Question

Q. Prove the following:

$$\sin \frac{\theta}{2} \sin \frac{7\theta}{2} + \sin \frac{3\theta}{2} \sin \frac{11\theta}{2} = \sin 2\theta \sin 5\theta$$

$$\text{Sol. } \sin \frac{\theta}{2} \sin \frac{7\theta}{2} + \sin \frac{3\theta}{2} \sin \frac{11\theta}{2}$$

$$\Rightarrow \frac{1}{2} \left(2\sin \frac{\theta}{2} \sin \frac{7\theta}{2} + 2\sin \frac{3\theta}{2} \sin \frac{11\theta}{2} \right)$$

$$\Rightarrow \frac{1}{2} \left\{ \cos \left(\frac{\theta}{2} - \frac{7\theta}{2} \right) - \cos \left(\frac{\theta}{2} + \frac{7\theta}{2} \right) + \cos \left(\frac{3\theta}{2} - \frac{11\theta}{2} \right) - \cos \left(\frac{3\theta}{2} + \frac{11\theta}{2} \right) \right\}$$

$$\Rightarrow \frac{1}{2} \{ \cos 3\theta - \cos 4\theta + \cos 4\theta - \cos 7\theta \}$$

$$\Rightarrow \frac{1}{2} \{ \cos 3\theta - \cos 7\theta \} = \frac{1}{2} \{ -2\sin 5\theta \sin(-2\theta) \}$$

$$= \sin 5\theta \sin 2\theta \quad \text{HHPP}$$

Acidic Question

Q. Find value of: $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

$$\text{Sol. } \tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$$

$$\Rightarrow \tan 9^\circ + \tan 81^\circ - (\tan 27^\circ + \tan 63^\circ)$$

$$\Rightarrow \frac{\sin 9^\circ}{\cos 9^\circ} + \frac{\sin 81^\circ}{\cos 81^\circ} - \left(\frac{\sin 27^\circ}{\cos 27^\circ} + \frac{\sin 63^\circ}{\cos 63^\circ} \right)$$

$$\Rightarrow \frac{\sin 9^\circ \sin 81^\circ + \cos 9^\circ \sin 81^\circ}{\cos 9^\circ \cos 81^\circ}$$

$$- \left(\frac{\sin 27^\circ \cos 63^\circ + \cos 27^\circ \sin 63^\circ}{\cos 27^\circ \cos 63^\circ} \right)$$

$$\Rightarrow \frac{\sin(9^\circ + 81^\circ)}{\cos 9^\circ \cdot \cos 81^\circ} - \frac{\sin(27^\circ + 63^\circ)}{\cos 27^\circ \cos 63^\circ}$$

$$\Rightarrow \frac{2}{2\cos 9^\circ \cdot \cos 81^\circ} - \frac{2}{2\cos 27^\circ \cos 63^\circ}$$

$$\Rightarrow \frac{2}{\cos 72^\circ} - \frac{2}{\cos 36^\circ} = 2 \left\{ \frac{\cos 36^\circ - \cos 72^\circ}{\cos 72^\circ \cos 36^\circ} \right\}$$

$$= 2 \left\{ \frac{+2\sin 54^\circ \sin(18^\circ)}{\cos 72^\circ \cos 36^\circ} \right\} = 4 \text{ fnf.}$$

Question [JEE-1984]

$$Q. \left(1 + \cos \frac{\pi}{8} \right) \left(1 + \cos \frac{3\pi}{8} \right) \left(1 + \cos \frac{5\pi}{8} \right) \left(1 + \cos \frac{7\pi}{8} \right)$$

is equal to

(A) $\frac{1}{2}$

(B) $\cos \frac{\pi}{8}$

(C) $\frac{1}{8}$

(D) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

$$\begin{aligned}
 \text{Sol. } & \left(1 + \cos \frac{\pi}{8}\right) \left(1 + \cos \frac{3\pi}{8}\right) \left(1 + \cos \frac{5\pi}{8}\right) \left(1 + \cos \frac{7\pi}{8}\right) \\
 & \Rightarrow \left(1 + \cos \frac{\pi}{8}\right) \left(1 + \cos \frac{3\pi}{8}\right) \left(1 - \cos \frac{3\pi}{8}\right) \left(1 - \cos \frac{\pi}{8}\right) \\
 & \Rightarrow \left(1 - \cos^2 \frac{\pi}{8}\right) \left(1 - \cos^2 \frac{3\pi}{8}\right) \\
 & \Rightarrow \sin^2 \frac{\pi}{8} \sin^2 \frac{3\pi}{8} = \left(\sin \frac{\pi}{8} \cdot \sin \frac{3\pi}{8}\right)^2 \\
 & \Rightarrow \left(\frac{2}{2} \sin \frac{\pi}{8} \cos \frac{\pi}{8}\right)^2 = \left(\frac{1}{2} \sin \frac{\pi}{4}\right)^2 = \left(\frac{1}{2} \cdot \frac{1}{\sqrt{2}}\right)^2 = \frac{1}{8} \text{ fnf.}
 \end{aligned}$$

Question [JEE-1984]

Q. If $\cos \alpha + \cos \beta = \frac{1}{2}$ & $\sin \alpha + \sin \beta = \frac{1}{3}$, then

find $\cos\left(\frac{\alpha+\beta}{2}\right)$, $\cos\left(\frac{\alpha-\beta}{2}\right)$, $\tan\left(\frac{\alpha+\beta}{2}\right)$ & $\tan\left(\frac{\alpha-\beta}{2}\right)$

$$\begin{aligned}
 \text{Sol. } & \cos \alpha + \cos \beta = \frac{1}{2} \\
 & \Rightarrow 2 \cos \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2} = \frac{1}{2} \quad \dots(i) \\
 & \Rightarrow \sin \alpha + \sin \beta = \frac{1}{3} \\
 & \Rightarrow 2 \sin \left(\frac{\alpha+\beta}{2}\right) \cos \left(\frac{\alpha-\beta}{2}\right) = \frac{1}{3} \quad \dots(ii) \\
 & (i) \div (ii) \Rightarrow \frac{2 \cos \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2} = \frac{1}{2}}{2 \sin \left(\frac{\alpha+\beta}{2}\right) \cos \left(\frac{\alpha-\beta}{2}\right) = \frac{1}{3}} = \frac{1}{3} \\
 & \Rightarrow \cot \left(\frac{\alpha+\beta}{2}\right) = +\frac{3}{2} \\
 & \Rightarrow \tan \left(\frac{\alpha+\beta}{2}\right) = \frac{2}{3} \text{ \& } \cos \left(\frac{\alpha+\beta}{2}\right) = \pm \frac{3}{\sqrt{13}} \\
 & \text{As: } \cos \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2} = \frac{1}{4} \\
 & \Rightarrow \pm \frac{3}{\sqrt{13}} \cos \left(\frac{\alpha-\beta}{2}\right) = \frac{1}{4} \\
 & \boxed{\cos \left(\frac{\alpha-\beta}{2}\right) = \pm \frac{\sqrt{13}}{12}}
 \end{aligned}$$

FORMULAS FOR MULTIPLE ANGLES



$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$\begin{aligned}
 \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\
 &= 2 \cos^2 \theta - 1 \\
 &= 1 - 2 \sin^2 \theta
 \end{aligned}$$

$$\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$\tan^2 \theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\begin{aligned}
 \sin 2\theta &= (\sin \theta + \cos \theta)^2 - 1 \\
 &= 1 - (\sin \theta - \cos \theta)^2
 \end{aligned}$$

Note

$$(1) \frac{1 + \cos 2\theta}{\sin 2\theta} = \cot \theta$$

$$(2) \frac{1 - \cos 2\theta}{\sin 2\theta} = \tan \theta$$

$$(3) \frac{1 + \sin 2\theta}{\cos 2\theta} = \tan \left[\frac{\pi}{4} + \theta\right]$$

$$(4) \frac{1 - \sin 2\theta}{\cos 2\theta} = \tan \left[\frac{\pi}{4} - \theta\right]$$

Q. If $\tan \theta = \frac{3}{4}$, $\pi < \theta < \frac{3\pi}{2}$, find value of

$\sin \frac{\theta}{2}$, $\cos \frac{\theta}{2}$, $\cos 2\theta$, $\tan 2\theta$

$$\text{Sol. } \pi < \theta < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$$

$$\text{If } \tan \theta = \frac{3}{4} \Rightarrow \cos \theta = -\frac{4}{5}$$

$$\Rightarrow 2 \cos^2 \frac{\theta}{2} - 1 = \frac{-4}{5} \Rightarrow \cos^2 \frac{\theta}{2} = \frac{1}{10}$$

$$\Rightarrow \cos \frac{\theta}{2} = \frac{1}{\sqrt{10}}$$

$$\text{Again, } \cos \theta = \frac{-4}{5} \Rightarrow 1 - 2 \sin^2 \left(\frac{\theta}{2}\right) = \frac{-4}{5}$$

$$\Rightarrow \sin^2 \frac{\theta}{2} = \frac{9}{10} \Rightarrow \sin \frac{\theta}{2} = \frac{3}{\sqrt{10}}$$

$$\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{7}{25}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \times \frac{3}{4}}{1 - \frac{9}{16}} = \frac{24}{7}$$

Acidic Question ★★

Q. If $\cot\left(\frac{\pi}{24}\right) = \sqrt{a} + \sqrt{b} + \sqrt{c} + \sqrt{d}$, where $(a, b, c, d \in \mathbb{N})$, then $a + b + c + d =$

Sol. $\frac{1 + \cos 2\theta}{\sin 2\theta} = \cot \theta \left(\theta = \frac{15^\circ}{2} \text{ or } \frac{\pi}{24} \right)$

$$\Rightarrow \frac{1 + \cos 15^\circ}{\sin 15^\circ} = \cot\left(\frac{15^\circ}{2}\right)$$

$$\Rightarrow \frac{1 + \frac{\sqrt{3} + 1}{2\sqrt{2}}}{\frac{\sqrt{3} - 1}{2\sqrt{2}}} = \cot\left(\frac{15^\circ}{2}\right)$$

$$\Rightarrow \cot\left(\frac{15^\circ}{2}\right) = \frac{(2\sqrt{2} + \sqrt{3} + 1) \times \frac{\sqrt{3} + 1}{2\sqrt{2}}}{\sqrt{3} - 1}$$

$$= \frac{2\sqrt{6} + 3 + \sqrt{3} + 2\sqrt{2} + \sqrt{3} + 1}{2}$$

$$= \frac{2\sqrt{6}}{2} + \frac{4}{2} + \frac{2\sqrt{3}}{2} + \frac{2\sqrt{2}}{2}$$

$$= \sqrt{6} + 2 + \sqrt{3} + \sqrt{2}$$

$$\cot\left(\frac{\pi}{24}\right) = \sqrt{6} + \sqrt{4} + \sqrt{3} + \sqrt{2}$$

$$\Rightarrow a + b + c + d = 6 + 4 + 3 + 2 = 15 \text{ fnf.}$$

FORMULAS FOR TRIPLE ANGLES

Formula for (3θ) :

(1) $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$

(2) $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$

(3) $\tan(3\theta) = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$

IMPORTANT VALUES (RAR)

Important values of memories:

(1) $\sin 18^\circ = \frac{\sqrt{5} - 1}{4} = \cos 72^\circ$

(2) $\sin 54^\circ = \frac{\sqrt{5} + 1}{4} = \cos 36^\circ$

(3) $\sin 36^\circ = \frac{\sqrt{10 - 2\sqrt{5}}}{4} = \cos 54^\circ$

(4) $\sin 72^\circ = \frac{\sqrt{10 + 2\sqrt{5}}}{4} = \cos 18^\circ$

(5) $\tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1 = \cot\left(\frac{3\pi}{8}\right)$

(6) $\tan\left(\frac{3\pi}{8}\right) = \sqrt{2} + 1 = \cot\left(\frac{\pi}{8}\right)$

CHALLENGER QUESTION

Q. If $\sin 5\theta = \sum_{r=0}^5 a_r \sin^r \theta$ then find $\sum_{r=0}^5 a_r$.

OR

If $\sin 5\theta$ is expanded in increasing power of $\sin \theta$, find all coefficient.

Sol. $\sin 5\theta = \sin(5\theta + 2\theta)$

$$\sin 5\theta = \sin 5\theta \cos 2\theta + \cos 3\theta \sin 2\theta$$

$$\Rightarrow \sin 5\theta = (3\sin\theta - 4\sin^3\theta)(1 - 2\sin^2\theta)$$

$$+ \cos^2\theta \{4\cos^2\theta - 3\} 2\sin\theta$$

Let: $\sin \theta = t$

$$\sin 5\theta = (3t - 4t^3)(1 - 2t^2) + (1 - t^2)(2t - 3t^3)$$

$$\Rightarrow \sin 5\theta = 16t^5 - 20t^3 + 5t$$

$$\Rightarrow \sin 5\theta = 16(\sin\theta)^5 - 20(\sin\theta)^3 + 5\sin\theta$$

$$\Rightarrow \text{Sum of all coeff.} = 16 - 20 + 5 = 1 \text{ fnf.}$$

Youtube Solution

$$\Rightarrow \sin 5\theta = \sum_{r=0}^5 a_r \sin^r \theta$$

$$\text{Put } \left(\theta = \frac{\pi}{2}\right): \sin \frac{5\pi}{2} = \sum_{r=0}^5 a_r = 1 \text{ fnf.}$$

Maha Bumper Question

Q. If $\alpha = \cot 70^\circ + 4 \cos 70^\circ$, $\beta = \sin^2 24^\circ - \cos^2 84^\circ$, $\gamma = \cos^2 73^\circ + \cos^2 47^\circ + \cos 73^\circ \cos 47^\circ$ and $\delta = \sin \frac{\pi}{5} \sin \frac{2\pi}{5} \sin \frac{3\pi}{5} \sin \frac{4\pi}{5}$. Then find value of $[\alpha]^2 + [8\beta] + [3\gamma] + [16\delta]$ (where $[.]$ represents G.I.F)

Sol. $\alpha = \frac{\cos 70^\circ}{\sin 70^\circ} + 4 \cos 70^\circ$

$$= \frac{\cos 70^\circ + 2(2 \cos 70^\circ \sin 70^\circ)}{\sin 70^\circ}$$

$$= \frac{\sin 20^\circ + 2 \sin 140^\circ}{\sin 70^\circ}$$

$$= \frac{(\sin 20^\circ + \sin 140^\circ) + \sin 140^\circ}{\sin 70^\circ}$$

$$= \frac{2 \sin 80^\circ \cos 60^\circ + \sin 140^\circ}{\sin 70^\circ}$$

$$\Rightarrow \alpha = \frac{2 \sin 70^\circ \cos 30^\circ}{\sin 70^\circ} = \sqrt{3}$$

$$\beta = \sin^2 24^\circ - \cos^2 84^\circ = \sin^2 24^\circ - \sin^2 6^\circ$$

$$= \sin(24^\circ + 6^\circ) \sin(24^\circ - 6^\circ)$$

$$\Rightarrow \beta = \frac{1}{2} \times \frac{\sqrt{5} - 1}{4} = \frac{\sqrt{5} - 1}{8}$$

$$\Rightarrow \gamma = \sin^2 17^\circ - \sin^2 47^\circ + 1 + \frac{1}{2}(2 \cos 73^\circ \cos 47^\circ)$$

$$= \sin(17^\circ + 47^\circ) \sin(-30^\circ) + 1$$

$$+ \frac{1}{2}(\cos 120^\circ + \cos 26^\circ)$$

$$1 = -\frac{\sin 64^\circ}{2} + 1 + \left(-\frac{1}{4}\right) + \frac{1}{2} \cos 26^\circ \Rightarrow \gamma = \frac{3}{4}$$

$$\Rightarrow \delta = \sin \frac{\pi}{5} \cdot \sin \frac{2\pi}{5} \cdot \sin \frac{2\pi}{5} \cdot \sin \frac{\pi}{5}$$

$$\delta = \left(\sin \frac{\pi}{5} \cdot \sin \frac{2\pi}{5}\right)^2$$

$$\delta = (\sin 36^\circ \sin 72^\circ)^2 = \left(\frac{\sqrt{10-2\sqrt{5}}}{4} \times \frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2$$

$$\delta = \left(\sqrt{\frac{100-20}{16}}\right)^2 = \left(\frac{\sqrt{80}}{16}\right)^2 = \frac{5}{16}$$

$$\Rightarrow [\alpha] + [8\beta] + [3\gamma] + [16\delta]$$

$$= [\sqrt{3}]^2 + [\sqrt{5} - 1] + \left[\frac{9}{4}\right] + [5] = 9 \text{ fnf.}$$

Question [JEE Main-2021] ★★★

Q. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$,

Then, $\sin x + \cos y$ is

- (A) $\frac{1+\sqrt{3}}{2}$ (B) $\frac{1-\sqrt{3}}{2}$
(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

Sol. $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$

$$2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) - \left(2 \cos^2\left(\frac{x+y}{2}\right) - 1\right) = \frac{3}{2}$$

मान लो: $\cos \frac{x+y}{2} = m, \cos \frac{x-y}{2} = n$

$$\Rightarrow 2mn - (2m^2 - 1) = \frac{3}{2}$$

$$\Rightarrow 2mn - 2m^2 - \frac{1}{2} = 0$$

$$\Rightarrow -n^2 - 4mn + 4m^2 + 1 + n^2 = 0$$

$$\Rightarrow 1 - n^2 + (n - 2m)^2 = 0$$

$$\left(\cos\left(\frac{x-y}{2}\right) - 2 \cos\left(\frac{x+y}{2}\right)\right)^2 + \sin^2\left(\frac{x-y}{2}\right) = 0$$

$$\Rightarrow \text{only possible when: } \sin\left(\frac{x-y}{2}\right) = 0 \Rightarrow (x=y)$$

$$\text{and } \left(\cos x = \frac{1}{2}\right) \Rightarrow x = y = 60^\circ$$

$$\Rightarrow \sin x + \cos y = \frac{\sqrt{3}+1}{2} \text{ fnf.}$$

CHALLENGER QUESTION [JEE Main-2023]

Q. $36(4\cos^2 9^\circ - 1)(4\cos^2 27^\circ - 1)(4\cos^2 81^\circ - 1)(4\cos^2 243^\circ - 1) =$

Sol. $(4\cos^2 9^\circ - 1) = 4(1 - \sin^2 9^\circ) - 1 = 3 - 4\sin^2 9^\circ$

$$= \frac{3 \sin 9^\circ - 4 \sin^3 9^\circ}{\sin 9^\circ} = \frac{\sin 27^\circ}{\sin 9^\circ}$$

$$36 \times \frac{\sin(27^\circ)}{\sin 9^\circ} \times \frac{\sin(81^\circ)}{\sin 27^\circ} \times \frac{\sin(243^\circ)}{\sin 81^\circ} \times \frac{\sin 729^\circ}{\sin 243^\circ}$$

$$\text{fnf} = 36$$

Question [JEE Main-2022]

Q. The value of $2\sin(12^\circ) - \sin(72^\circ)$ is

- (A) $\frac{\sqrt{5}(1-\sqrt{3})}{4}$ (B) $\frac{1-\sqrt{5}}{8}$
(C) $\frac{\sqrt{3}(1-\sqrt{5})}{2}$ (D) $\frac{\sqrt{3}(1-\sqrt{5})}{4}$

Sol. $2\sin(12^\circ) - \sin(72^\circ)$
 $= \sin 12^\circ + (\sin 12^\circ - \sin 72^\circ)$
 $= \sin 12^\circ + 2\cos 42^\circ \left(-\frac{1}{2}\right)$
 $= \sin 12^\circ - \sin 48^\circ = 2\cos 30^\circ \sin(-18^\circ)$
 $= 2 \times \frac{\sqrt{3}}{2} \times \left\{-\left(\frac{\sqrt{5}-1}{4}\right)\right\} = \frac{\sqrt{3}(1-\sqrt{5})}{4} \text{ fnf.}$

IMPORTANT RESULTS

- (1) $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
 $= \cos^2 B - \cos^2 A$
 (2) $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$
 $= \cos^2 B - \sin^2 A$

IMPORTANT RESULTS

- (1) $\sin \theta \sin(60^\circ + \theta) \sin(60^\circ - \theta) = \frac{1}{4} \sin(3\theta)$
 (2) $\cos \theta \cos(60^\circ + \theta) \cos(60^\circ - \theta) = \frac{1}{4} \cos(3\theta)$
 (3) $\tan \theta \tan(60^\circ + \theta) \tan(60^\circ - \theta) = \tan(3\theta)$

Example

Q. Find value of $\sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ =$

Sol. $\sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ =$
 $= \frac{\sin 6^\circ \sin(60^\circ + 6^\circ) \sin(60^\circ - 6^\circ)}{\sin 54^\circ}$
 $\times \frac{\sin 18^\circ \sin(60^\circ + 18^\circ) \sin(60^\circ - 18^\circ)}{\sin 18^\circ}$
 $= \frac{\frac{1}{4} \sin 18^\circ \frac{1}{4} \sin(54^\circ)}{\sin 54^\circ \sin 18^\circ} = \frac{1}{16} \text{ fnf.}$

Question [JEE Main-2022]

Q. If $\sin^2(10^\circ)\sin(20^\circ)\sin(40^\circ)\sin(50^\circ)\sin(70^\circ)$

$= \alpha - \frac{1}{16}$ then $16 + \alpha^{-1}$ is equal to.

Sol. LHS

$= (\sin 10^\circ \sin 50^\circ \sin 70^\circ) \left(\frac{\sin 10^\circ}{2} 2 \sin 20^\circ \sin 40^\circ\right)$
 $= \left(\frac{1}{4} \sin 30^\circ\right) \times \frac{1}{2} \sin 10^\circ (\cos 20^\circ - \cos 60^\circ)$
 $= \frac{1}{4} \times \frac{1}{2} \times \frac{1}{2} \times \sin 10^\circ \left(\cos 20^\circ - \frac{1}{2}\right)$
 $= \frac{1}{16} \left(\frac{1}{2} (2 \sin 10^\circ \cos 20^\circ) - \frac{1}{2} \sin 10^\circ\right)$
 $= \frac{1}{16} \left(\frac{1}{2} (\sin 30^\circ - \cos 10^\circ) - \frac{1}{2} \sin 10^\circ\right)$
 $= \frac{1}{16} \left(\frac{1}{4} - \sin 10^\circ\right) = \frac{1}{64} - \frac{1}{16} \sin 10^\circ$
 $\Rightarrow \alpha = \frac{1}{64} \Rightarrow 16 + \alpha^{-1} = 16 + 64 = 80 \text{ fnf.}$

Bumper Question

Q. If $A = \theta$, $B = \theta + \frac{\pi}{3}$ & $C = \theta - \frac{\pi}{3}$, then show that.

(i) $\tan \theta + \tan\left(\theta + \frac{\pi}{3}\right) + \tan\left(\theta - \frac{\pi}{3}\right) = 3 \tan 3\theta$

(ii) $\tan \theta \tan\left(\theta + \frac{\pi}{3}\right) + \tan \theta \tan\left(\theta - \frac{\pi}{3}\right)$
 $+ \tan\left(\theta + \frac{\pi}{3}\right) \tan\left(\theta - \frac{\pi}{3}\right) = -3$

(iii) $\tan \theta \tan\left(\theta + \frac{\pi}{3}\right) \tan\left(\theta - \frac{\pi}{3}\right) = -\tan 3\theta$

Sol. (i) $\text{LHS} = \tan \theta + \left(\frac{\tan \theta + \sqrt{3}}{1 - \sqrt{3} \tan \theta} + \frac{\tan \theta - \sqrt{3}}{1 + \sqrt{3} \tan \theta}\right)$
 $\frac{\tan \theta + \sqrt{3} \tan^2 \theta + 3 \tan \theta + \tan \theta - \sqrt{3} - \sqrt{3} \tan^2 \theta + 3 \tan \theta}{(1 - 3 \tan^2 \theta)}$

$\text{LHS} = \tan \theta + \frac{8 \tan \theta}{1 - 3 \tan^2 \theta}$
 $= \frac{9 \tan \theta - 3 \tan^3 \theta}{1 - 3 \tan^2 \theta} = 3 \tan 3\theta \quad \text{HHPP}$

$$\begin{aligned}
 \text{(ii)} \quad & \left\{ A = 0, B = 0 + \frac{\pi}{3}, C = 0 - \frac{\pi}{3} \right\} \\
 \Rightarrow & A + B + C = 3\theta \\
 \Rightarrow & \tan(A + B + C) = \tan 3\theta \\
 \Rightarrow & \frac{S_1 - S_3}{1 - S_2} = \tan 3\theta \\
 \Rightarrow & \frac{3 \tan 3\theta - (-\tan 3\theta)}{1 - S_2} = \tan 3\theta \\
 \Rightarrow & \frac{4 \tan 3\theta}{1 - S_2} = \tan 3\theta \\
 4 = & 1 - S_2 \Rightarrow S_2 = -3 \\
 \text{(iii)} \quad & \text{DIBY}
 \end{aligned}$$

PRODUCT OF COSINE WITH DOUBLE ANGLES

$$\cos \theta \cdot \cos 2\theta \cdot \cos 4\theta \cdot \cos 8\theta \dots \cos(2^{n-1}\theta) = \frac{\sin 2^n \theta}{2^n \sin \theta}$$

where $\theta \equiv$ smallest angle & $n \equiv$ number of terms

Example

$$Q. \cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{3\pi}{7} = \underline{\hspace{2cm}}$$

$$\begin{aligned}
 \text{Sol. } & \cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{3\pi}{7} \\
 &= \left(-\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} \right) = \frac{-\sin \left(\frac{2^3 \pi}{7} \right)}{2^3 \sin \left(\frac{\pi}{7} \right)} \\
 &= \frac{-\sin \left(\frac{8\pi}{7} \right)}{8 \sin \frac{\pi}{7}} = \frac{-\sin \left(\pi + \frac{\pi}{7} \right)}{8 \sin \frac{\pi}{7}} = \frac{1}{8} \text{ fnf.}
 \end{aligned}$$

Question

$$Q. \text{ The value of } \cos \frac{\pi}{2^2} \cdot \cos \frac{\pi}{2^3} \dots \cos \frac{\pi}{2^{10}} \sin \frac{\pi}{2^{10}} \text{ is}$$

$$\begin{aligned}
 \text{Sol. } & \underbrace{\cos \frac{\pi}{2^2} \cdot \cos \frac{\pi}{2^3} \dots \cos \frac{\pi}{2^{10}}}_{n=9, \theta = \frac{\pi}{2^{10}}} \sin \frac{\pi}{2^{10}} \\
 & n = 9, \theta = \frac{\pi}{2^{10}}
 \end{aligned}$$

$$= \frac{\sin \left(2^9 \cdot \frac{\pi}{2^{10}} \right)}{2^9 \cdot \sin \frac{\pi}{2^{10}}} \times \sin \frac{\pi}{2^{10}} = \frac{1}{2^9} = \frac{1}{512} \text{ fnf.}$$

Question [JEE Main-2022]

$$Q. 96 \sin \left(\frac{\pi}{33} \right) \cos \left(\frac{2\pi}{33} \right) \sin \left(\frac{4\pi}{33} \right) \sin \left(\frac{8\pi}{33} \right) \sin \left(\frac{16\pi}{33} \right) \text{ is equal to}$$

$$\begin{aligned}
 \text{Sol. } & 2 \sin \left(\frac{\pi}{22} \right) \sin \left(\frac{3\pi}{22} \right) \sin \left(\frac{5\pi}{22} \right) \sin \left(\frac{7\pi}{22} \right) \sin \left(\frac{9\pi}{22} \right) \\
 &= 2 \cdot \cos \frac{2\pi}{22} \cdot \cos \frac{4\pi}{22} \cdot \cos \frac{8\pi}{22} \left(-\cos \frac{16\pi}{22} \right) \left(-\sin \frac{32\pi}{22} \right) \\
 \theta = & \frac{2\pi}{22}, n = 5 \\
 &= 2 \times \frac{\sin \left(2^5 \cdot \frac{2\pi}{22} \right)}{2^5 \sin \left(\frac{2\pi}{22} \right)} = \frac{1}{16} \text{ fnf.}
 \end{aligned}$$

Question [JEE Main-2023]

$$Q. 96 \sin \left(\frac{\pi}{33} \right) \cos \left(\frac{2\pi}{33} \right) \sin \left(\frac{4\pi}{33} \right) \sin \left(\frac{8\pi}{33} \right) \sin \left(\frac{16\pi}{33} \right) \text{ is equal to}$$

$$\text{Sol. } 3 \text{ DIBY}$$

TRIGONOMETRIC SERIES

100%
Question
aayega!!

KKPNC

Type O1: Sine & cosine series with angle in A.P.

$$(i) \sin(\alpha) + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \dots$$

$$+ \sin(\alpha + (n-1)\beta) = \frac{\sin \left(\frac{n\beta}{2} \right)}{\sin \left(\frac{\beta}{2} \right)} \sin \left(\left(\alpha + (n-1) \frac{\beta}{2} \right) \right)$$

$$(ii) \cos(\alpha) + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \dots$$

$$+ \cos(\alpha + (n-1)\beta) = \frac{\sin \left(\frac{n\beta}{2} \right)}{\sin \left(\frac{\beta}{2} \right)} \sin \left(\left(\alpha + (n-1) \frac{\beta}{2} \right) \right)$$

Where $n =$ Total terms

$\alpha =$ First angle

$\beta =$ Common difference of angles

Question

Q. The value of $[100(x - 1)]$ is where $[x]$ is the greatest integer less than or equal to x and

$$x = \frac{\sum_{n=1}^{44} \cos n^\circ}{\sum_{n=1}^{44} \sin n^\circ}$$

(A) 140 (B) 141 (C) 142 (D) 144

Sol. $N^r = \cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \dots + \cos 44^\circ$

$$= \frac{\sin 44 \times \frac{1}{2^\circ}}{\sin \frac{1}{2^\circ}} \cdot \cos \left(1^\circ + 43 \times \frac{1}{2^\circ} \right)$$

$$D^r = \sin 1^\circ + \sin 2^\circ + \dots + \sin 44^\circ$$

$$= \frac{\sin 44 \times \frac{1}{2^\circ}}{\sin \frac{1}{2^\circ}} \cdot \sin \left(1^\circ + 43 \times \frac{1}{2^\circ} \right)$$

$$x = \cot \left(\frac{45^\circ}{2} \right) = \cot \frac{\pi}{8} = \sqrt{2} + 1$$

$$[100(x - 1)] = [100\sqrt{2}] = [100 \times 1.414] = 141 \text{ fnf.}$$

CHALLENGER QUESTION

Q. Let $S = \sum_{r=1}^{90} (2r) \sin(2r)^\circ$ then find of S is (are)

(A) $\cot 1^\circ$ (B) $\tan 1^\circ$
(C) $\frac{\cos^2 20^\circ - \sin^2 19^\circ}{\sin^2 26^\circ - \sin^2 25^\circ}$ (D) $\sqrt{\frac{1 - \cos 2^\circ}{1 + \cos 2^\circ}}$

Sol. $S = 2\sin 2^\circ + 4\sin 4^\circ + 6\sin 6^\circ + 8\sin 8^\circ + \dots + 176\sin 176^\circ + 178\sin 178^\circ + 180\sin 180^\circ$
 $\Rightarrow S = 180\sin 2^\circ + 180\sin 4^\circ + 180\sin 6^\circ + \dots + 180\sin 88^\circ + 90$

$$S = 180\{\sin 2^\circ + \sin 4^\circ + \sin 6^\circ + \dots + \sin 88^\circ\} + 90$$

$$S = 180 \times \frac{\sin 44 \times 1^\circ}{\sin 1^\circ} \cdot \sin(2^\circ + 43 \times 1^\circ)$$

$$S = \left(180 \times \frac{\sin 44^\circ}{\sin 1^\circ} \cdot \sin 45^\circ \right) + 90; \text{Ans. [AC]}$$

Reducible to Type-01:

Power हटाओ $\rightarrow$ Answer पाओ

Q. $\sum_{r=0}^n \sin^2 \frac{r\pi}{n}$ is equal to

(A) $\frac{n+1}{2}$ (B) $\frac{n-1}{2}$
(C) $\frac{n}{2} - 1$ (D) $\frac{n}{2}$

Sol. $\sum_{r=0}^n \sin^2 \frac{r\pi}{n} = \sin^2 0 + \sin^2 \frac{\pi}{n} + \sin^2 \frac{2\pi}{n} + \dots + \sin^2 \frac{n\pi}{n}$
 $= \frac{1}{2} \left(1 - \cos \frac{2\pi}{n} \right) + \frac{1}{2} \left(1 - \cos \frac{4\pi}{n} \right) + \frac{1}{2} \left(1 - \cos \frac{6\pi}{n} \right) + \dots + \frac{1}{2} \left(1 - \cos \frac{2n\pi}{n} \right)$
 $= n \times \frac{1}{2} - \frac{1}{2} \left\{ \cos \frac{2\pi}{n} + \cos \frac{4\pi}{n} + \cos \frac{6\pi}{n} + \dots \right\} = \frac{n}{2} \text{ fnf.}$

Question

Q. $S = \cos^3 \theta + \cos^3 2\theta + \cos^3 3\theta + \dots$ upto n -terms

Sol. $S = \cos^3 \theta + \cos^3 2\theta + \cos^3 3\theta + \dots$ upto n -terms

$$S = \frac{\cos 3\theta + 3\cos \theta}{4} + \frac{\cos 6\theta + 3\cos 2\theta}{4} + \frac{\cos 9\theta + 3\cos 3\theta}{4} + \dots$$

$$S = \frac{1}{4} \{ \cos 3\theta + \cos 6\theta + \cos 9\theta + \dots \} + \frac{3}{4} \{ \cos \theta + \cos 2\theta + \cos 3\theta + \dots \}$$

now apply direct formula.

Type-02: Splitting T_n as difference of two terms (Viral + Trending)



IMPORTANT NOTE

Denominator में product of trigonometric terms

$$\begin{aligned} \frac{1}{\sin A \sin B} &= \frac{\sin(B-A)}{\sin(B-A) \sin A \sin B} \\ &= \frac{1}{\sin(B-A)} \left\{ \frac{\sin B \cos A}{\sin A \sin B} - \frac{\cos B \sin A}{\sin A \sin B} \right\} \\ &= \frac{1}{\sin(B-A)} \{ \cot A - \cot B \} \\ \frac{1}{\cos A \cos B} &= \frac{\sin(B-A)}{\sin(B-A) \cos A \cos B} \\ &= \frac{1}{\sin(B-A)} \{ \tan B - \tan A \} \end{aligned}$$

Question

Q. The sum of the series $\sum_{r=0}^{88} \sec r^\circ \cdot \sec(r+1)^\circ$ is equal to

$$\text{Sol. } T_{r+1} = \frac{1}{\cos r^\circ \cos(r+1)^\circ} = \frac{\sin((r+1)^\circ - r^\circ)}{\sin 1^\circ (\cos r^\circ \cos(r+1)^\circ)}$$

$$T_{r+1} = \frac{1}{\sin 1^\circ} \{\tan(r+1)^\circ - \tan r^\circ\}$$

$$T_1 = \frac{1}{\sin 1^\circ} \{\tan 1^\circ - \tan 0^\circ\}$$

$$T_2 = \frac{1}{\sin 1^\circ} \{\tan 2^\circ - \tan 1^\circ\}$$

$$T_3 = \frac{1}{\sin 1^\circ} \{\tan 3^\circ - \tan 2^\circ\}$$

$$\vdots$$

$$T_{89} = \frac{1}{\sin 1^\circ} \{\tan 89^\circ - \tan 88^\circ\}$$

$$\Rightarrow \text{Sum} = \frac{1}{\sin 1^\circ} (\tan 89^\circ - \tan 0^\circ)$$

$$= \frac{1}{\sin 1^\circ} \frac{\sin 89^\circ}{\cos 89^\circ}$$

$$\text{sum} = \frac{1}{\sin 1^\circ} \frac{\cos 1^\circ}{\sin 1^\circ} = (\operatorname{cosec} 1^\circ \cot 1^\circ) \text{fnf.}$$

Question [JEE(Adv.)-2016]

Q. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right) \sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to

- (A) $3 - \sqrt{3}$ (B) $2(3 - \sqrt{3})$
(C) $2(\sqrt{3} - 1)$ (D) $2(2 + \sqrt{3})$

$$\text{Sol. } T_r = \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right) \sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$$

$$T_r = \frac{1}{\sin \frac{\pi}{6}} \left(\frac{\sin(B-A)}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right) \sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)} \right)$$

$$\text{where; } A = \frac{\pi}{4} + \frac{k\pi}{6} - \frac{\pi}{6}, B = \frac{\pi}{4} + \frac{k\pi}{6}$$

$$\Rightarrow B - A = \frac{\pi}{6}$$

$$\Rightarrow T_r = \cot\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)$$

$$T_1 = \cot\left(\frac{\pi}{4}\right) - \cot\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

$$T_2 = \cot\left(\frac{\pi}{4} + \frac{\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{2\pi}{6}\right)$$

$$\vdots$$

$$T_{13} = \cot\left(\frac{\pi}{4} + \frac{12\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{13\pi}{6}\right)$$

$$\text{Sum} = \cot \frac{\pi}{4} - \cot \frac{15\pi}{12} = (\sqrt{3} - 1)$$

$$\text{Sum} = \frac{1}{\sin \frac{\pi}{6}} (\sqrt{3} - 1) = 2(\sqrt{3} - 1) \text{fnf.}$$

Question [IIT-JEE-2009]

Q. For $0 < \theta < \frac{\pi}{2}$, then solution (s) of

$$\sum_{m=1}^6 \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right) \operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2} \text{ is}$$

$$\text{Sol. } \left(\frac{\pi}{12}, \frac{5\pi}{12}\right) \text{ DIBY}$$

IMPORTANT SERIES INITIATORS

$$(1) \cot \theta - \tan \theta = \cot 2\theta$$

$$(2) \cot\left(\frac{\theta}{2}\right) - \cot \theta = \operatorname{cosec} \theta$$

Prove that:

$$(1) \cot \theta - \tan \theta = \cot 2\theta$$

$$\text{LHS} = \frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} = \left(\frac{2(\cos^2 \theta - \sin^2 \theta)}{2 \cos \theta \sin \theta} \right)$$

$$= \frac{2 \cos 2\theta}{\sin 2\theta} = 2 \cot 2\theta$$

$$(2) \cot(\theta/2) - \cot \theta = \operatorname{cosec} \theta$$

$$\text{LHS} = \frac{\cos \frac{\theta}{2}}{\sin \frac{\theta}{2}} - \frac{\cos \theta}{\sin \theta}$$

$$= \frac{\sin \theta \cdot \cos \frac{\theta}{2} - \cos \theta \cdot \sin \frac{\theta}{2}}{\sin \frac{\theta}{2} \sin \theta} = \operatorname{cosec} \theta$$

Question

Q. Show that $\sum_{r=0}^n \operatorname{cosec}(2^r x) = \cot \frac{x}{2} - \cot 2^n x$

Sol. $T_1 = \operatorname{cosec}(x) = \cot \frac{x}{2} - \cot x$

$T_2 = \operatorname{cosec}(2x) = \cot x - \cot 2x$

$T_3 = \operatorname{cosec}(4x) = \cot 2x - \cot 4x$

$\vdots$

$T_n = \operatorname{cosec}(2^n x) = \operatorname{cosec}(2^{n-1} x) - \cot(2^n x)$

Add-----

$\cot \frac{x}{2} - \cot 2^n x = \sum_{r=0}^n \operatorname{cosec}(2^r x) \quad \text{HHPP}$

Question

Q. Prove that

$$\sum_{r=0}^n \frac{\sin(3^r \theta)}{\cos(3^{r+1} \theta)} = \frac{\tan(3^{n+1} \theta) - \tan \theta}{2}$$

Sol. $T_1 = \frac{2 \sin \theta}{\cos 3\theta} \times \frac{\cos \theta}{2 \cos \theta} = \frac{\sin 2\theta}{2 \cos 3\theta \cdot \cos \theta}$

$= \frac{1}{2} (\tan 3\theta - \tan \theta)$

$T_2 = \frac{2 \sin 3\theta}{\cos 9\theta} \times \frac{\cos 3\theta}{2 \cos 3\theta} = \frac{\sin 6\theta}{2 \cos 9\theta \cdot \cos 3\theta}$

$= \frac{1}{2} (\tan 9\theta - \tan 3\theta)$

$\vdots$

$T_{n+1} = \frac{\sin 3^n \theta}{\cos(3^{n+1} \theta)} \dots = \frac{1}{2} (\tan(3^{n+1} \theta) - \tan 3^n \theta)$

$T_1 + T_2 + \dots + T_{n+1} = \frac{1}{2} (\tan(3^{n+1} \theta) - \tan \theta) \quad \text{HHPP}$

Acidic Question

Q. Show that:

$$\sum_{r=0}^n \frac{1}{2^r} \tan\left(\frac{\theta}{2^r}\right) = \frac{1}{2^n} \cot\left(\frac{\theta}{2^n}\right) - 2 \cot 2\theta$$

Sol. $r = 0 \Rightarrow \tan \theta = \cot \theta - 2 \cot 2\theta$

$r = 1 \Rightarrow \frac{1}{2} \tan \frac{\theta}{2} = \frac{1}{2} \left(\cot \frac{\theta}{2} \right) - (\cot \theta)$

$r = 2 \Rightarrow \frac{1}{2^2} \tan \frac{\theta}{2^2} = \frac{1}{2^2} \left(\cot \frac{\theta}{2^2} \right) - \left(\frac{1}{2} \cot \frac{\theta}{2} \right)$

$\vdots$

$r = n \Rightarrow \frac{1}{2^n} \tan \frac{\theta}{2^n} = \frac{1}{2^n} \left(\cot \frac{\theta}{2^n} - 2 \cot \frac{\theta}{2^{n-1}} \right)$

$\Rightarrow$ Adding all we get $= \frac{1}{2^n} \cot \frac{\theta}{2^n} - 2 \cot 2\theta$

HHPP

CONDITIONAL IDENTITIES

Involving sine & Cosine

(1) $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

(2) $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$

(3) $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

(4) $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

Question

Q. Show that: If $A + B + C = \pi$ then $\sin^2 A + \sin^2 B + \sin^2 C = 2(1 + \cos A \cos B \cos C)$

Sol. $\sin^2 A + \sin^2 B + \sin^2 C$

$= \frac{1 - \cos 2A}{2} + \frac{1 - \cos 2B}{2} + \frac{1 - \cos 2C}{2}$

$= \frac{1}{2} \{3 - (\cos 2A + \cos 2B + \cos 2C)\}$

$= \frac{1}{2} \{3 + 1 + 4 \cos A \cos B \cos C\}$

$= 2(1 + \cos A \cos B \cos C) \quad \text{HHPP}$

Involving Tangent & Co-tangent

(1) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

(2) $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

(3) $\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$

(4) $\left(\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} \right) = 1$

Question

Q. If $\cos^{-1} a + \cos^{-1} b + \cos^{-1} c = \pi$ then the value of $(a\sqrt{1-a^2} + b\sqrt{1-b^2} + c\sqrt{1-c^2})$ will be

(A) $2(\sqrt{1-a^2}\sqrt{1-b^2}\sqrt{1-c^2})$

(B) $\sqrt{1-a^2}\sqrt{1-b^2}\sqrt{1-c^2}$

(C) abc

(D) $2abc$

Sol. $A + B + C = \pi$

$\cos^{-1} a + \cos^{-1} b + \cos^{-1} c = \pi$

$\cos^{-1} a = A \Rightarrow a = \cos A \Rightarrow \sin A = \sqrt{1-a^2}$

$$\begin{aligned}\cos^{-1}a &= B \Rightarrow b = \cos B \Rightarrow \sin B = \sqrt{1-b^2} \\ \cos^{-1}c &= C \Rightarrow c = \cos C \Rightarrow \sin C = \sqrt{1-c^2} \\ \text{Req.} &= \frac{2\cos A \sin A + 2\cos B \sin B + 2\cos C \sin C}{2} \\ &= \frac{\sin 2A + \sin 2B + \sin 2C}{2} = \frac{4 \sin A \sin B \sin C}{2} \\ &= 2\sqrt{1-a^2}\sqrt{1-b^2}\sqrt{1-c^2} \text{ fnf.}\end{aligned}$$

Question

Q. If $xy + yz + zx = 1$ where $x, y, z \in \mathbb{R}^+$ then prove that

$$\frac{x}{1+x^2} + \frac{y}{1+y^2} + \frac{z}{1+z^2} = \frac{2}{\sqrt{(1+x^2)(1+y^2)(1+z^2)}}$$

Sol. $x = \tan \frac{A}{2}, y = \tan \frac{B}{2}, z = \tan \frac{C}{2}$

$$A + B + C = \pi$$

$$\text{LHS} = \sin A + \sin B + \sin C$$

$$= 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$$

$$\text{RHS} = \frac{2 \times 2}{\sqrt{(1+x^2)(1+y^2)(1+z^2)}}$$

$$= \frac{4}{\sqrt{\sec^2 \frac{A}{2} \sec^2 \frac{B}{2} \sec^2 \frac{C}{2}}} = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$$

HHPP

TRIANGLE INEQUALITIES

(1) For $\triangle ABC$: $1 < \cos A + \cos B + \cos C \leq \frac{3}{2}$

Also, $0 < \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \leq \frac{1}{8}$

(2) For $\triangle ABC$: $0 < \sin A + \sin B + \sin C \leq \frac{3\sqrt{3}}{2}$

Also, $0 < \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2} \leq \frac{3\sqrt{3}}{8}$

(3) For acute angle $\triangle ABC$:

$$\tan A + \tan B + \tan C \geq 3\sqrt{3}$$

(4) For $\triangle ABC$: $-1 < \cos A \cos B \cos C \leq \frac{1}{8}$

Also, $0 < \sin A \sin B \sin C \leq \frac{3\sqrt{3}}{8}$

Proof:

(1) $\cos A + \cos B + \cos C = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} + \cos C$

$$= 2 \sin \frac{C}{2} \cos \left(\frac{A-B}{2} \right) + 1 - 2 \sin^2 \frac{C}{2} \leq 2 \sin \frac{C}{2} + 1 - 2 \sin^2 \frac{C}{2}$$

$$\Rightarrow \cos A + \cos B + \cos C \leq \frac{3}{2} - 2 \left(\sin \frac{C}{2} - \frac{1}{2} \right)^2$$

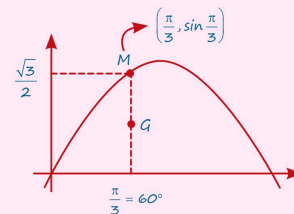
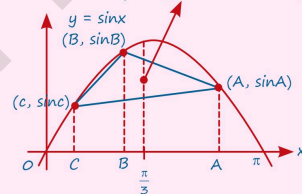
$$\Rightarrow \cos A + \cos B + \cos C \leq \frac{3}{2} \text{ when } \sin \frac{C}{2} = \frac{1}{2}$$

and $\cos \frac{A-B}{2} = 1$

$\Rightarrow$ max. value occurs when triangle is equilateral.

(2) Jensen's inequality:

$$G \equiv \left(\frac{A+B+C}{3}, \frac{\sin A + \sin B + \sin C}{3} \right)$$



$\Rightarrow$ y-co-ord. of $M >$ y-co-ord of G

$$\frac{\sqrt{3}}{2} > \frac{\sin A + \sin B + \sin C}{3}$$

$\Rightarrow$ Equality will hold for equilateral $\triangle$.

(3) Acute $\Rightarrow \left. \begin{matrix} \tan A \\ \tan B \\ \tan C \end{matrix} \right\} \rightarrow \boxed{+ve}$

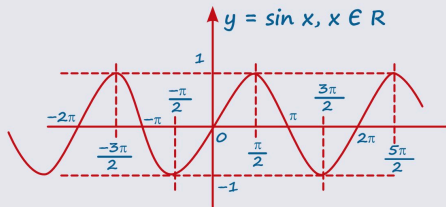
Apply: $AM \geq GM$

$$\Rightarrow \frac{\tan A + \tan B + \tan C}{3} \geq (\tan A \tan B \tan C)^{\frac{1}{3}}$$

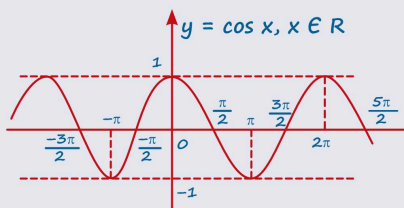
$$\Rightarrow \frac{x}{3} \geq (x)^{\frac{1}{3}}; \text{ (cube both sides) } \frac{x^3}{27} \geq x$$

$$\Rightarrow x^2 \geq 27 \Rightarrow \boxed{x \geq 3\sqrt{3}}$$

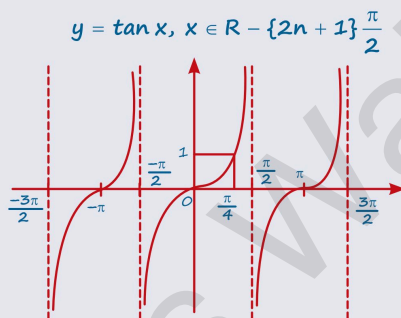
GRAPH OF TRIGONOMETRIC FUNCTION



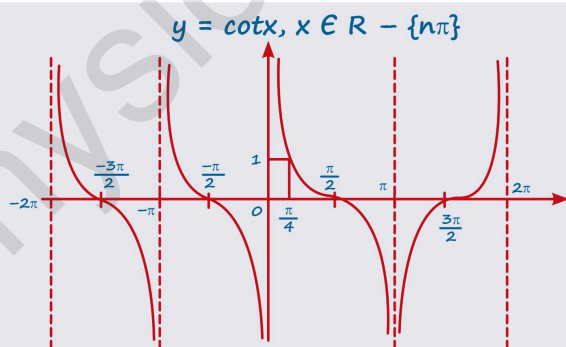
Range $(\sin x) \in [-1, 1]$
Period = 2π



Range $(\cos x) \in [-1, 1]$
Period = 2π

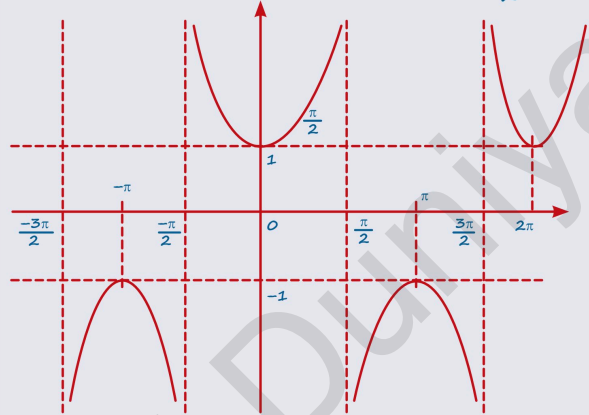


Range $\in (-\infty, \infty) \equiv \mathbb{R}$



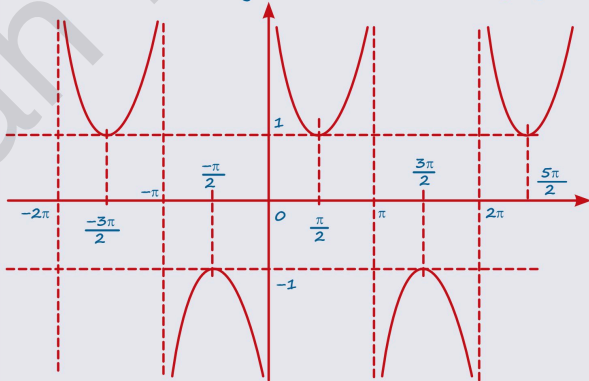
Range $\in (-\infty, \infty) \equiv \mathbb{R}$
Period = π

$$y = \sec x, x \in \mathbb{R} - \{2n + 1\} \frac{\pi}{2}$$



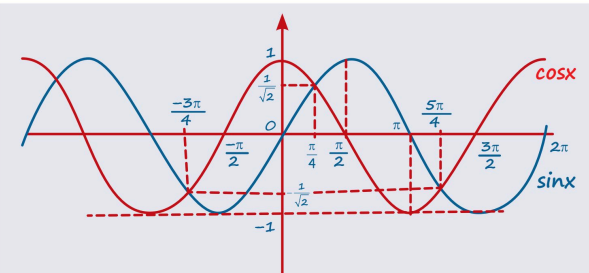
Range $\in (-\infty, -1] \cup [1, \infty)$
Period = 2π

$$y = \operatorname{cosec} x, x \in \mathbb{R} - \{n\pi\}$$

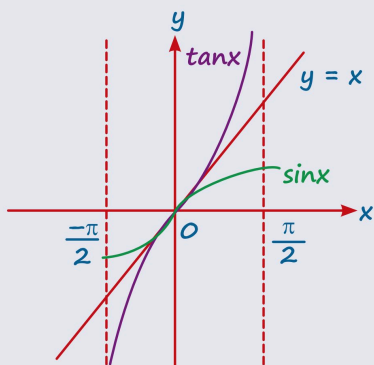


Range $\in (-\infty, -1] \cup [1, \infty)$
Period = 2π

GRAPH OF SINE & COSINE ON SAME AXIS



GRAPH OF SINE & TAN ON SAME AXIS



$$x \in \left(0, \frac{\pi}{2}\right) \Rightarrow \sin x < x < \tan x$$

$$x \in \left(-\frac{\pi}{2}, 0\right) \Rightarrow \sin x > x > \tan x$$

RANGE OF TRIGONOMETRIC FUNCTIONS

Type-O1: Based on Range of standard trigonometric Functions

$$\sin x \in [-1, 1] \quad \tan x \in \mathbb{R}$$

$$\cos x \in [-1, 1] \quad \cot x \in \mathbb{R}$$

$$\sec x \in (-\infty, -1] \cup [1, \infty)$$

$$\operatorname{cosec} x \in (-\infty, -1] \cup [1, \infty)$$

Question

Q. Find range of following functions:

(i) $y = 3\sin x - 5$

(ii) $y = \frac{1}{5\cos x + 2}$

Sol. (i) $y = 3\sin x - 5$

$$\Rightarrow \sin x \in [-1, 1]$$

$$\Rightarrow 3\sin x \in [-3, 3]$$

$$\Rightarrow 3\sin x - 5 \in [-8, -2] \text{ fnf.}$$

(ii) $\Rightarrow \cos x \in [-1, 1] \Rightarrow 5\cos x \in [-5, 5]$

$$\Rightarrow 5\cos x + 2 \in [-3, 7]$$

$$\Rightarrow \frac{1}{5\cos x + 2} \in \left(-\infty, -\frac{1}{3}\right] \cup \left[\frac{1}{7}, \infty\right) \text{ fnf.}$$

Type-O2: Range by completing perfect square

Question

Q. Find range of: $y = \cos^2 x - 4\cos x + 13$

Sol. $y = \cos^2 x - 4\cos x + 13$

$$y = \cos^2 x - 2(2)\cos x + 4 + 9$$

$$y = (\cos x - 2)^2 + 9$$

$$\cos x \in [-1, 1] \Rightarrow \cos x - 2 \in [-3, -1]$$

$$\Rightarrow (\cos x - 2)^2 \in [1, 9]$$

$$\Rightarrow (\cos x - 2)^2 + 9 \in [10, 18] \text{ fnf.}$$

Question

Q. Find range of: $y = \sin^4 x + \cos^4 x$

Sol. $y = \sin^4 x + \cos^4 x = 1 - 2(\sin x \cos x)^2$

$$= 1 - 2\left(\frac{\sin 2x}{2}\right)^2 = 1 - \frac{2}{4}\sin^2 2x$$

$$y = 1 - \frac{1}{2}\sin^2 2x \Rightarrow y \in \left[\frac{1}{2}, 1\right] \text{ fnf.}$$

Type-O3: Form: $f(x) = a\cos x \pm b\sin x$

Range: $[-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$



Proof:

$$f(x) = a\cos x \pm b\sin x$$

Multiply & divide by $(\sqrt{a^2 + b^2})$:

$$f(x) = \sqrt{a^2 + b^2} \left(\frac{a}{\sqrt{a^2 + b^2}} \cos x \pm \frac{b}{\sqrt{a^2 + b^2}} \sin x \right)$$

$$y = \sqrt{a^2 + b^2} \cos(x \pm \alpha)$$

$$y = \sqrt{a^2 + b^2} \times [-1, 1]$$

$$y \in [-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$$

Question

Q. Find the maximum & minimum value of:

$$y = 3\cos\left(x + \frac{\pi}{3}\right) + 5\cos x + 3$$

Sol. $y = 3 \left\{ \cos x \cdot \cos \frac{\pi}{3} - \sin x \cdot \sin \frac{\pi}{3} \right\} + 5 \cos x + 3$
 $= \frac{3}{2} \cos x - \frac{3\sqrt{3}}{2} \sin x + 5 \cos x + 3$
 $= \frac{13}{2} \cos x - \frac{3\sqrt{3}}{2} \sin x + 3$
 $= [-7, 7] + 3 = [-4, 10] \text{ fnf.}$

Question

Q. Find the maximum & minimum value of

(i) $y = \log_2 \left(\frac{3 \sin x - 4 \cos x + 15}{10} \right)$

(ii) $y = 27^{\cos 3x} \cdot 81^{\sin 3x}$

Sol. (i) $y = \log_2 \left(\frac{3 \sin x - 4 \cos x + 15}{10} \right)$
 $\alpha = 3 \sin x - 4 \cos x \in [-5, 5]$
 $\alpha + 15 \in [10, 20] \Rightarrow \frac{\alpha + 15}{10} \in [1, 2]$
 $\log_2 \left(\frac{\alpha + 15}{10} \right) \in [\log_2 1, \log_2 2] \in [0, 1] \text{ fnf.}$
(ii) $y = 27^{\cos 3x} \cdot 81^{\sin 3x}$
 $\Rightarrow y = 3^{3 \cos 3x} \cdot 3^{4 \sin 3x} \Rightarrow y = 3^{3 \cos 3x + 4 \sin 3x}$
where: $(3 \cos 3x + 4 \sin 3x) \in [-5, 5]$
 $\Rightarrow y \in [3^{-5}, 3^5] \text{ fnf.}$

Type-04: **(RAR)**

(1) $a^2 \tan^2 \theta + b^2 \cot^2 \theta \geq 2ab$

(2) $a^2 \sec^2 \theta + b^2 \csc^2 \theta \geq (a + b)^2$

(where a and b are +ve real numbers)

Proof:

(1) $a^2 \tan^2 \theta + b^2 \frac{1}{\tan^2 \theta} \Big|_{\min}$

Apply: AM $\geq$ GM

$\Rightarrow \frac{a^2 \tan^2 \theta + b^2 \cot^2 \theta}{2} \geq (a^2 b^2)^{\frac{1}{2}}$

$\Rightarrow a^2 \tan^2 \theta + b^2 \cot^2 \theta \geq 2ab$

(2) $a^2 \sec^2 \theta + b^2 \csc^2 \theta \geq (a + b)^2$

LHS = $a^2 \sec^2 \theta + b^2 \csc^2 \theta$

= $a^2 (\tan^2 \theta + 1) + b^2 (\cot^2 \theta + 1)$

= $a^2 \tan^2 \theta + a^2 + b^2 \cot^2 \theta + b^2 \geq (a + b)^2$

Critical Thinking Question

Q. $y = 18 \sec^2 x + 8 \cos^2 x$, find minimum value of y & also find corresponding value of x .

Sol. Sunita: $y = 18 \sec^2 x + 8 \cos^2 x$

$y = 8 \cos^2 x + \frac{18}{\cos^2 x}$

AM $\geq$ GM

$\frac{8 \cos^2 x + \frac{18}{\cos^2 x}}{2} \geq (8 \times 18)^{\frac{1}{2}}$

$8 \cos^2 x + 18 \sec^2 x \geq 3 \times 4 \times 2$

24 wrong ans.

Equality hold: $8 \cos^2 x = \frac{18}{\cos^2 x}$

$\Rightarrow \cos^4 x = \frac{18}{8} \Rightarrow \cos^4 x = \frac{9}{4}$

$\Rightarrow \cos^2 x = \frac{3}{2} \Rightarrow \text{Not Possible.}$

Sushmita: $y = 8 \cos^2 x + 8 \sec^2 x + 10 \sec^2 x$

$y = 8 \left(\cos^2 x + \frac{1}{\cos^2 x} \right) + 10 \sec^2 x$
 $(\geq 2) \quad (\geq 1)$

$\Rightarrow$ at $(x = 0)$ we get:

$y_{\min} = 8 \times 2 + 10 \times 1 = 26 \text{ fnf.}$

Question

Q. If $x^2 + y^2 = 4$, then find the maximum value of $\frac{x^3 + y^3}{x + y}$.

Sol. Put: $x = 2 \cos \theta, y = 2 \sin \theta$

$\alpha = \frac{8 \cos^3 \theta + 8 \sin^3 \theta}{2(\cos \theta + \sin \theta)}$

= $\frac{4(\cos \theta + \sin \theta)(\cos^2 \theta + \sin^2 \theta - \cos \theta \sin \theta)}{(\cos \theta + \sin \theta)}$

$\alpha = 4 \left(1 - \frac{\sin 2\theta}{2} \right)$

$\alpha \in [2, 6] \text{ fnf.}$

Question

Q. For all $x, y \in \mathbb{R}$, find the range of

$\frac{(x + y)(1 - xy)}{(1 + x^2)(1 + y^2)}$

Sol. $\frac{(x+y)(1-xy)}{(1+x^2)(1+y^2)}$

Put: $\{x = \tan \theta \text{ \& } y = \tan \alpha\}$

$$\frac{(\tan \theta + \tan \alpha)(1 - \tan \theta \tan \alpha)}{(1 + \tan^2 \theta)(1 + \tan^2 \alpha)}$$

$$= \frac{2 \left(\frac{\sin(\theta + \alpha)}{\cos \theta \cos \alpha} \cdot \frac{\cos(\theta + \alpha)}{\cos \theta \cdot \cos \alpha} \right)}{2 \left(\frac{1}{\cos^2 \theta} \right) \left(\frac{1}{\cos^2 \alpha} \right)}$$

$$\Rightarrow \frac{\sin(2\theta + 2\alpha)}{2}$$

max. $\frac{1}{2}$
min. $-\frac{1}{2}$

Miscellaneous Questions

Question

Q. If $f(\theta) = \sin^3 \theta + \sin^3 \left(\theta + \frac{2\pi}{3} \right) + \sin^3 \left(\theta + \frac{4\pi}{3} \right)$

then the value of $f\left(\frac{\pi}{18}\right) + f\left(\frac{7\pi}{18}\right)$ is equal to

(A) $\frac{3}{4}$

(B) $\frac{1}{4}$

(C) $\frac{1}{8}$

(D) 0

Sol. $a = \sin \theta$,

$$b = \sin \left(\theta + \frac{2\pi}{3} \right)$$

$$c = \sin \left(\theta + \frac{4\pi}{3} \right)$$

$$= \sin \left(\theta + \pi - \frac{\pi}{3} \right)$$

$$= \sin \left(\theta + \pi + \frac{\pi}{3} \right)$$

$$= \sin \left(\pi - \left(\frac{\pi}{3} - \theta \right) \right)$$

$$c = -\sin \left(\theta + \frac{\pi}{3} \right)$$

$$b = \sin \left(\frac{\pi}{3} - \theta \right)$$

$$\Rightarrow a + b + c = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

$$f(\theta) = 3(a)(b)(c)$$

$$= 3 \sin \theta \cdot \sin \left(\frac{\pi}{3} - \theta \right) \left(-\sin \left(\frac{\pi}{3} + \theta \right) \right)$$

$$f(\theta) = -3 \left(\frac{1}{4} \sin 3\theta \right)$$

$$f\left(\frac{\pi}{18}\right) = -\frac{3}{4} \sin \left(\frac{\pi}{6} \right)$$

$$f\left(\frac{7\pi}{18}\right) = -\frac{3}{4} \sin \left(\frac{7\pi}{6} \right) = \frac{3}{4} \sin \left(\frac{\pi}{6} \right)$$

$$f\left(\frac{\pi}{18}\right) + f\left(\frac{7\pi}{18}\right) = 0 \text{ f.n.f.}$$

Critical Thinking Question

Q. Show:

$$\left(\tan^2 \frac{\pi}{7} + \tan^2 \frac{2\pi}{7} + \tan^2 \frac{3\pi}{7} \right) \cdot \left(\cot^2 \frac{\pi}{7} + \cot^2 \frac{2\pi}{7} + \cot^2 \frac{3\pi}{7} \right) = 105$$

Sol. If $\theta = \frac{\pi}{7} \Rightarrow 7\theta = \pi \Rightarrow 4\theta = \pi - 3\theta$

$$\tan(4\theta) = \tan(\pi - 3\theta)$$

$$\tan(4\theta) = -\tan(3\theta)$$

$$\Rightarrow \frac{4 \tan \theta - 4 \tan^3 \theta}{1 - 6 \tan^2 \theta + \tan^4 \theta} = - \left(\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right)$$

मान लो: $\tan \theta = t = \tan \frac{\pi}{7}$

$$\Rightarrow \frac{4t(1-t^2)}{1-6t^2+t^4} = \frac{-t(3-t^2)}{1-3t^2}$$

$$t^6 - 21t^4 + 35t^2 - 7 = 0$$

$$(t^2)^3 - 21(t^2)^2 + 35(t^2) - 7 = 0$$

put: $\alpha = t^2$

$$\alpha^3 - 21\alpha^2 + 35\alpha - 7 = 0$$

$$\alpha_1 = \tan^2 \frac{\pi}{7}; \alpha_2 = \tan^2 \frac{2\pi}{7}; \alpha_3 = \tan^2 \frac{3\pi}{7}$$

$$\text{Sum} = \frac{-(-21)}{1} = 21$$

$$\alpha_3 - 21\alpha^2 + 35\alpha - 7 = 0$$

Now replace: $\alpha \rightarrow \frac{1}{\alpha}$

$$\frac{1}{\alpha^3} - \frac{21}{\alpha^2} + \frac{35}{\alpha} - 7 = 0$$

$$\frac{1 - 21\alpha + 35\alpha^2 - 7\alpha^3}{\alpha^3} = 0$$

$$7\alpha^3 - 35\alpha^2 + 21\alpha - 1 = 0$$

$$\text{Sum} = \frac{-(-35)}{7} = 5$$

$$\begin{aligned} & \cot^2 \frac{\pi}{7} \\ & \cot^2 \frac{2\pi}{7} \\ & \cot^2 \frac{3\pi}{7} \end{aligned}$$

Question

Q. The maximum value of

$$\cos^2(\cos 33\pi + \theta) + \sin^2(\sin(45\pi + \theta)) \text{ is}$$

(A) $1 + \sin^2 1$

(B) 2

(C) $1 + \cos^2 1$

(D) $\cos^2 2$

Sol. $\cos^2(\cos(33\pi + \theta)) + \sin^2(\sin(45\pi + \theta))$

$$= (\cos(-\cos \theta))^2 + \sin^2(-\sin \theta)^2$$

$$= \cos^2(\cos \theta) + \sin^2(\sin \theta)$$

$$\Rightarrow \cos^2(\cos \theta) \leq 1 \Rightarrow \sin^2(\sin \theta) \leq \sin^2(1)$$

add-----

$$\cos^2(\cos \theta) + \sin^2(\sin \theta) \leq 1 + \sin^2 1 \text{ fnf.}$$

Question [JEE Main-2025]

Q. If for $\theta \in \left[-\frac{\pi}{3}, 0\right]$, the points

$$(x, y) = \left(3 \tan\left(\theta + \frac{\pi}{3}\right), 2 \tan\left(\theta + \frac{\pi}{6}\right)\right)$$

lie on $xy + \alpha x + \beta y + \gamma = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to:

(A) 75

(B) 96

(C) 80

(D) 72

Sol. Let $\phi = \theta + \frac{\pi}{3} \Rightarrow \theta = \phi - \frac{\pi}{6}$

$$x = 3 \tan\left(\theta + \frac{\pi}{3}\right) = 3 \tan\left(\phi + \frac{\pi}{6}\right)$$

$$y = 2 \tan \phi$$

$$\tan\left(\phi + \frac{\pi}{6}\right) = \frac{\tan \phi + \frac{1}{\sqrt{3}}}{1 - \tan \phi \cdot \frac{1}{\sqrt{3}}}$$

$$\frac{x}{3} = \frac{\frac{y}{2} + \frac{1}{\sqrt{3}}}{1 - \frac{y}{2} \cdot \frac{1}{\sqrt{3}}} \Rightarrow x = \frac{3y(\sqrt{3} + 2)}{2\sqrt{3} - y}$$

$$xy + \alpha x + \beta y + \gamma = 0$$

$$3 \left(\frac{y\sqrt{3} + 2}{2\sqrt{3} - y} \right) y + \alpha \left(3 \frac{(y\sqrt{3} + 2)}{2\sqrt{3} - y} \right) + \beta y + \gamma = 0$$

$$\Rightarrow (3\sqrt{3} - \beta)y^2 + (6 + 3\sqrt{3}\alpha + 2\sqrt{3}\beta - \gamma)y$$

$$+ (6\alpha + 2\sqrt{3}\gamma) = 0$$

For this identity to hold for all θ , coefficients must be 0

$$\therefore \beta = 3\sqrt{3}$$

$$\gamma = -\alpha\sqrt{3}$$

$$6 + 3\sqrt{3}\alpha + (2\sqrt{3})(3\sqrt{3}) + \alpha\sqrt{3} = 0$$

$$\Rightarrow \alpha = -2\sqrt{3}$$

$$\Rightarrow \gamma = 6$$

$$\alpha^2 + \beta^2 + \gamma^2 = 75 \text{ fnf}$$

Question [JEE Main-2025]

Q. If $10\sin^4\theta + 15\cos^4\theta = 6$, then the value of

$$\frac{27 \operatorname{cosec}^6\theta + 8 \sec^6\theta}{16 \sec^8\theta} \text{ is}$$

(A) $\frac{2}{5}$

(B) $\frac{3}{5}$

(C) $\frac{1}{5}$

(D) $\frac{3}{4}$

Sol. $10\sin^4\theta + 15\cos^4\theta = 6$

$$\Rightarrow 10\sin^4\theta + 10\cos^4\theta + 5\cos^4\theta = 6$$

$$\Rightarrow 10[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta]$$

$$+ 5\cos^4\theta = 6$$

$$\Rightarrow 10 - 20(1 - \cos^2\theta)\cos^2\theta + 5\cos^4\theta = 6$$

$$\text{Let } \cos^2\theta = x$$

$$10 - 20(x - x^2) + 5x^2 = 6$$

$$\Rightarrow 25x^2 - 20x + 4 = 0$$

$$(5x - 2)^2 = 0 \Rightarrow x = \frac{2}{5}$$

$$\Rightarrow \cos^2\theta = \frac{2}{5} \Rightarrow \sin^2\theta = \frac{3}{5},$$

$$\sec^2\theta = \frac{5}{2} \Rightarrow \operatorname{cosec}^2\theta = \frac{5}{3}$$

$$\frac{27 \operatorname{cosec}^6\theta + 8 \sec^6\theta}{16 \sec^8\theta} = \frac{27 \left(\frac{5}{3}\right)^3 + 8 \left(\frac{5}{2}\right)^3}{16 \left(\frac{5}{2}\right)^4}$$

$$= \frac{5^3 + 5^3}{5^4} = \frac{2(5^3)}{5^4} = \frac{2}{5} \text{ fnf}$$

Question [JEE Main-2025]

Q. If $\sum_{r=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + (r-1)\frac{\pi}{6}\right) \sin\left(\frac{\pi}{4} + \frac{r\pi}{6}\right)} = a\sqrt{3} + b,$

$a, b \in \mathbb{Z}$, then $a^2 + b^2$ is equal to:

- (A) 10 (B) 4
(C) 8 (D) 2

Sol.
$$\frac{1}{\sin \frac{\pi}{6}} \sum_{r=1}^{13} \frac{\sin \left[\left(\frac{\pi}{4} + \frac{r\pi}{6} \right) - \left(\frac{\pi}{4} + (r-1)\frac{\pi}{6} \right) \right]}{\sin \left(\frac{\pi}{4} + (r-1)\frac{\pi}{6} \right) \sin \left(\frac{\pi}{4} + \frac{r\pi}{6} \right)}$$
$$= \frac{1}{\sin \frac{\pi}{6}} \sum_{r=1}^{13} \left(\cot \left(\frac{\pi}{4} + (r-1)\frac{\pi}{6} \right) - \cot \left(\frac{\pi}{4} + \frac{r\pi}{6} \right) \right)$$
$$= 2\sqrt{3} - 2 = a\sqrt{3} + b$$

So, $a^2 + b^2 = 8.$

selection के बाद एक डिब्बा मिठाई

उसके घर भी भिजवानी है,

कभी हम उसके थे,

आज दुनिया हमारी दिवानी है।

GENERAL SOLUTIONS OF T.E.

Types of Solutions:

General solution: All angles

Principle solution: $\theta \in [0, 2\pi)$

Particular solution: All solutions in the given interval.

Note

$$\sin x = 0 \Leftrightarrow x = n\pi$$

$$\cos x = 0 \Leftrightarrow x = (2n+1)\frac{\pi}{2}$$

$$\sin x = 1 \Leftrightarrow x = (4n+1)\frac{\pi}{2}$$

$$\sin x = -1 \Leftrightarrow x = (4n-1)\frac{\pi}{2}$$

$$\cos x = 1 \Leftrightarrow x = 2n\pi$$

$$\cos x = -1 \Leftrightarrow x = (2n-1)\pi$$

(Where 'n' is any integer)

ONLY FOUR FORMULA'S

$$(1) \sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, (n \in \mathbb{I})$$

$$(2) \cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha, (n \in \mathbb{I})$$

$$(3) \tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha, (n \in \mathbb{I})$$

$$(4) \sin^2 \theta = \sin^2 \alpha \Rightarrow \theta = n\pi \pm \alpha, (n \in \mathbb{I})$$

$$\cos^2 \theta = \cos^2 \alpha$$

$$\tan^2 \theta = \tan^2 \alpha$$

Proof:

$$\sin \theta = \sin \alpha \Rightarrow \sin \theta - \sin \alpha = 0$$

$$2 \sin \left(\frac{\theta - \alpha}{2} \right) \cos \left(\frac{\theta + \alpha}{2} \right) = 0$$

$$\sin \left(\frac{\theta - \alpha}{2} \right) = 0 \Rightarrow \frac{\theta - \alpha}{2} = n\pi \Rightarrow \theta = 2n\pi + \alpha$$

$$\theta = -4\pi + \alpha, 2\pi + \alpha, 4\pi + \alpha, \alpha, -2\pi + \alpha, \dots$$

$$\text{Or } \cos \left(\frac{\theta + \alpha}{2} \right) = 0 \Rightarrow \frac{\theta + \alpha}{2} = (2n+1)\frac{\pi}{2}$$

$$\theta = (2n+1)\pi - \alpha$$

$$\pi - \alpha, 3\pi - \alpha, 5\pi - \alpha, -\pi - \alpha, \dots$$

$$\theta = n\pi + (-1)^n \alpha, n \in \mathbb{I}$$

Proof:

$$\cos \theta = \cos \alpha \Rightarrow \cos \theta - \cos \alpha = 0$$

$$-2 \sin \left(\frac{\theta + \alpha}{2} \right) \sin \left(\frac{\theta - \alpha}{2} \right) = 0$$

$$\theta = 2n\pi - \alpha \text{ (or) } \theta = 2n\pi + \alpha$$

$$\Rightarrow \theta = 2n\pi \pm \alpha$$

Question

Q. Solve following equation: $\tan 3\theta = -1$

$$\text{Sol. Sumit: } \tan 3\theta = \tan \left(-\frac{\pi}{4} \right)$$

$$3\theta = n\pi - \frac{\pi}{4} = \frac{\pi(4n-1)}{4}$$

$$\theta = (4n-1)\frac{\pi}{12} \quad \text{Same}$$

$$\text{Santosh: } \tan 3\theta = \tan \frac{3\pi}{4}$$

$$3\theta = n\pi + \frac{3\pi}{4} \Rightarrow \theta = (4n+3)\frac{\pi}{12}$$

Question

Q. Solve following equation: $\cos 3\theta = \cos \theta$

$$\text{Sol. } \cos 3\theta = \cos \theta$$

$$3\theta = 2n\pi \pm \theta$$

Taking (+) sign

$$3\theta = 2n\pi + \theta$$

$$2\theta = 2n\pi$$

Taking (-) sign

$$3\theta = 2n\pi - \theta$$

$$4\theta = 2n\pi$$

$$\theta = n\pi$$

$$\theta = \frac{n\pi}{2}$$

$$0, \pi, 2\pi, \dots$$

$$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \dots$$

$$-\pi, -2\pi, \dots$$

$$-\frac{\pi}{2}, -\pi, -\frac{3\pi}{2}, \dots$$

Union: $\theta = \frac{n\pi}{2}, n \in \mathbb{I} \text{ fnf.}$

Question

Q. Solve following equation: $\sin 9\theta = \sin \theta$

Sol. $9\theta = n\pi + (-1)^n \theta$

If $n \equiv \text{even}$

$$9\theta = n\pi + \theta \Rightarrow 8\theta = n\pi$$

$$\theta = \frac{n\pi}{8}, n \in \text{even}; \text{ put } n = 2m, m \in \mathbb{I}$$

$$\theta = \frac{2m\pi}{8} = \frac{m\pi}{4}$$

If $n \equiv \text{odd}$

$$9\theta = n\pi - \theta \Rightarrow 10\theta = n\pi$$

$$\theta = \frac{n\pi}{10}, n \in \text{odd}; \text{ put } n = 2m + 1, m \in \mathbb{I}$$

$$\theta = (2m + 1) \frac{\pi}{10}$$

$$\text{fnf} \Rightarrow \theta \in \left\{ \frac{m\pi}{4} \right\} \cup \left\{ (2m + 1) \frac{\pi}{10} \right\}, m \in \mathbb{I}.$$

Question [JEE (Adv.)-2018]

Q. Let a, b, c be three non zero numbers such that the equation $\sqrt{3}a \cos x + 2b \sin x = c$,

$x \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$ has two distinct real roots

α and β with $\alpha + \beta = \frac{\pi}{3}$. Then the value of

$\frac{b}{a}$ is

Sol. **Sunita:** $\sqrt{3}a \cos \alpha + 2b \sin \alpha = c \quad \dots(1)$

$$\sqrt{3}a \cos \beta + 2b \sin \beta = c \quad \dots(2)$$

Subtract

$$\sqrt{3}a \{\cos \alpha - \cos \beta\} + 2b \{\sin \alpha - \sin \beta\} = 0$$

$$\sqrt{3}a \left(-2 \sin \frac{(\alpha - \beta)}{2} \sin \frac{(\alpha + \beta)}{2} \right) + 2b \left(2 \cos \frac{(\alpha + \beta)}{2} \sin \frac{(\alpha - \beta)}{2} \right) = 0$$

$$-\sqrt{3}a \times \frac{1}{2} + 2b \frac{\sqrt{3}}{2} = 0$$

$$\sqrt{3}b = \frac{\sqrt{3}a}{2} \Rightarrow \frac{b}{a} = \frac{1}{2} \text{ fnf.}$$

Sushmita:

$$\sqrt{3}a \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) + 2b \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) = c$$

मान लो: $\tan \frac{x}{2} = t$

$$\sqrt{3}a - \sqrt{3}at^2 + 4bt = c + ct^2$$

$$(c + \sqrt{3}a)t^2 - 4bt + c - \sqrt{3}a = 0$$

$\tan \frac{\alpha}{2}$
 $\tan \frac{\beta}{2}$

$$\tan \left(\frac{\alpha}{2} + \frac{\beta}{2} \right) = \frac{1}{\sqrt{3}}.$$

$$\Rightarrow \frac{\tan \frac{\alpha}{2} + \tan \frac{\beta}{2}}{1 - \tan \frac{\alpha}{2} \tan \frac{\beta}{2}} = \frac{1}{\sqrt{3}} = \frac{-\left(\frac{-4b}{c + \sqrt{3}a} \right)}{1 - \left(\frac{c - \sqrt{3}a}{c + \sqrt{3}a} \right)}$$

$$\Rightarrow \frac{b}{a} = \frac{1}{2} \text{ fnf}$$

Question

Q. Solve following equation:

$$(2 \sin \theta - \sqrt{3})^2 + (\tan \theta + \sqrt{3})^2 = 0$$

Sol. **Sunita:** $\sin \theta = \frac{\sqrt{3}}{2}$ & $\tan \theta = -\sqrt{3}$

$$\sin \theta = \sin \frac{\pi}{3}$$

$$\theta = n\pi + (-1)^n \frac{\pi}{3} \quad \dots(1)$$

$$\theta = \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{-4\pi}{3}, \frac{-5\pi}{3}, \dots \text{ and } \tan \theta = -\sqrt{3}$$

$$\tan \theta = \tan \left(-\frac{\pi}{3} \right) \Rightarrow \theta = m\pi - \frac{\pi}{3} \quad \dots(2)$$

$$\theta = -\frac{\pi}{3}, \frac{2\pi}{3}, -\frac{4\pi}{3}, \frac{5\pi}{3}, \frac{8\pi}{3}.$$

$\Rightarrow$ from (1) $\cap$ (2)

$$\theta = -\frac{4\pi}{3}, \frac{2\pi}{3}, \frac{8\pi}{3}, \frac{14\pi}{3}, \dots$$

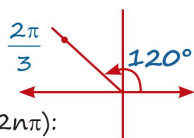
$$\theta = (6n - 4)\frac{\pi}{3}, n \in \mathbb{I} \text{ fnf.}$$

Sushmita: $\sin \theta = \frac{\sqrt{3}}{2} \cap \tan \theta = -\sqrt{3}$

$$\Rightarrow \theta = \frac{2\pi}{3}$$

$\Rightarrow$ Now to Generalise add $(2n\pi)$:

$$\theta = 2n\pi + \frac{2\pi}{3} = (6n + 2)\frac{\pi}{3}, n \in \mathbb{I} \text{ fnf.}$$



DHYAAN RKHNA

For whole Mathematics

1. **Avoid squaring:** In case of squaring final solution be checked.

Squaring se extra solutions aa skte hai.

2. **Avoid cancellation in multiplication.**

Cancellation leads to loss of original solutions.

$$\text{If } ab = bc \nRightarrow a = c$$

$$\Rightarrow b(a - c) = 0 \Rightarrow (a = c) \cup (b = 0)$$

$$b = 0 \text{ (or) } \cup a - c = 0$$

$$\text{If } \frac{b}{a} = \frac{c}{a} \Rightarrow b = c$$

$$\text{If } a + b = a + c \Rightarrow b = c$$

3. **Answer shouldn't contain values for which any of given function is not defined.**

(tan; cot; cosec; sec) अगर में present हो तो सतर्क रहे सावधान रहें।

Ex.: Solve: $\tan 3x - \tan 2x = 1 + \tan 3x \tan 2x$

Sol. $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$

$$\tan (3x - 2x) = 1 = \tan \frac{\pi}{4} \Rightarrow x = n\pi + \frac{\pi}{4}$$

$$\left. \begin{aligned} x &= \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \dots \\ &-\frac{3\pi}{4}, -\frac{7\pi}{4}, \dots \end{aligned} \right\} \tan 2x, \text{ not defined for all}$$

so $x \in \phi$ (No solution)

Question

Q. Solve: $\tan x \tan 4x = 1$

Sol. $\frac{1}{\cot x} \tan 4x = 1$

$$\tan 4x = \cot x \Rightarrow \tan 4x = \tan \left(\frac{\pi}{2} - x \right)$$

$$4x = n\pi + \frac{\pi}{2} - x \Rightarrow 5x = \frac{2n\pi + \pi}{2}$$

$$x = (2n + 1)\frac{\pi}{10}, n \in \mathbb{I} \text{nt.}$$

$$x = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10}, \frac{11\pi}{10}, \frac{13\pi}{10}, \frac{15\pi}{10}, \dots$$

$$\frac{-\pi}{10}, \frac{-3\pi}{10}, \dots$$

$$x \neq (2n + 1)\frac{\pi}{2} \text{ \& } x \neq (2n + 1)\frac{\pi}{8}$$

$$\text{fnf} \Rightarrow x \in \left\{ (2n + 1)\frac{\pi}{10} \right\} - \left\{ (2n + 1)\frac{\pi}{2} \right\}, n \in \mathbb{I}$$

4. **Aapka answer, dikhne mein alag ho skta hai.**

Ex: $\sin x = -\frac{1}{2}$

$$\sin x = \sin \left(-\frac{\pi}{6} \right) \text{ or } \sin x = \sin \frac{7\pi}{6}$$

$$x = n\pi + (-1)^{n+1} \frac{\pi}{6} \text{ or } x = n\pi + (-1)^n \frac{7\pi}{6}$$

both are Same

Note

Sin, cos, tan $\rightarrow$ +ve value then $\alpha \in \text{I quad.}$

Sin $\rightarrow$ -ve value then take $\alpha \in \text{IV quad.}$

cos $\rightarrow$ -ve value then take $\alpha \in \text{II quad.}$

tan $\rightarrow$ -ve value then take $\alpha \in \text{IV quad.}$

THOUGHT PROCESS

Try to make function and argument (angle) same.

Factorize into linear factors and write general solution.

Agr factorization na ho to perfect square banane ka socho.

Look for an identity.

TYPES OF TRIGONOMETRIC EQNS



Type-01

Solution of trigonometric equation by factorization or equation which are expressed in quadratic form.

Question

Q. Solve $(2 \sin x - \cos x)(1 + \cos x) = \sin^2 x$ in $[0, 2\pi]$.

Sol. $(2 \sin x - \cos x)(1 + \cos x) - (1 - \cos x)(1 + \cos x) = 0$

$$(1 + \cos x) \{2 \sin x - \cos x - 1 + \cos x\} = 0$$

$$\cos x = -1 \text{ (or) } \sin x = \frac{1}{2}$$

$$\Rightarrow x = \pi, \frac{\pi}{6}, \frac{5\pi}{6} \text{ fnf.}$$

Q. Find the number of solution of the equation $16 \sin^2 x + 16 \cos^2 x = 10$, where $0 \leq x \leq 2\pi$.

Sol. मान लो: $16 \sin^2 x = t$

$$\Rightarrow 16 \cos^2 x = \frac{16}{t}$$

$$\Rightarrow t + \frac{16}{t} = 10$$

$$\Rightarrow t^2 - 10t + 16 = 0$$

$$\Rightarrow (t - 8)(t - 2) = 0$$

$$\Rightarrow t = 2, 8$$

$$\text{As } (t = 8) \Rightarrow 2^{4 \sin^2 x} = 2^3$$

$$\Rightarrow \sin^2 x = \frac{3}{4} \Rightarrow \sin x = \pm \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$\text{Also: } (t = 2) \Rightarrow 2^{4 \sin^2 x} = 2^1$$

$$\Rightarrow \sin^2 x = \frac{1}{4} \Rightarrow \sin x = \pm \frac{1}{2}$$

$$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

No. of solutions = 8 fnf.

Question [JEE Main-2022]

Q. Let $S = \{\theta \in [-\pi, \pi] - \left\{\pm \frac{\pi}{2}\right\} : \sin \theta \tan \theta$

$+ \tan \theta = \sin 2\theta\}$ If $T = \sum_{\theta \in S} \cos 2\theta$, then $T + n(S)$ is equal.

(A) $7 + \sqrt{3}$

(B) 9

(C) $8 + \sqrt{3}$

(D) 10

Sol. $\tan \theta (\sin \theta + 1) - \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) = 0$

$$\tan \theta \left\{ \sin \theta + 1 - \frac{2}{1 + \tan^2 \theta} \right\} = 0$$

$$\tan \theta \{ \sin \theta + 1 - 2(1 - \sin^2 \theta) \} = 0$$

$$\tan \theta (2 \sin^2 \theta + \sin \theta - 1) = 0$$

$$\tan \theta (2 \sin \theta - 1)(\sin \theta + 1) = 0$$

$$\tan \theta = 0 \cup \sin \theta = \frac{1}{2} \cup \sin \theta = -1$$

$$\theta = -\pi, 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$$

$$n(S) = 5 \text{ \& } T = 4 \Rightarrow T + n(S) = 9 \text{ fnf.}$$

Question [JEE Main-2024]

Q. If $2 \sin^3 x + \sin 2x \cos x + 4 \sin x - 4 = 0$ has exactly 3 solutions in the interval $\left[0, \frac{n\pi}{2}\right]$,

$n \in \mathbb{N}$, then the roots of the equation $x^2 + nx + (n - 3) = 0$ belong to

(A) $(0, \infty)$

(B) $(-\infty, 0)$

(C) $\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$

(D) $\mathbb{Z}$

Sol. $2 \sin^3 x + 2 \sin x (1 - \sin^2 x) + 4 \sin x = 4$

$$\Rightarrow 6 \sin x = 4$$

$$\Rightarrow \sin x = \frac{2}{3} \Rightarrow 3 \text{ solutions in } \left[0, \frac{5\pi}{2}\right]$$

$$\text{So, } n = 5 \Rightarrow x^2 + 5x + 2 = 0$$

$$x = \frac{-5 \pm \sqrt{25 - 4 \times 2}}{2}$$

$$x = \frac{-5 \pm \sqrt{17}}{2} \text{ both } (-ve) \Rightarrow (-\infty, 0) \text{ fnf.}$$

Type-02

Solving trigonometric equations by introducing an Auxiliary argument. Equation of the form of $a \cos \theta + b \sin \theta = c$.

Ex. Find general value of: $\sqrt{3} \cos x + \sin x = 2$

$$\begin{aligned} \text{Sol. } \Rightarrow \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x &= 1 \\ \Rightarrow \cos \left(x - \frac{\pi}{6} \right) &= \cos 0 \Rightarrow x - \frac{\pi}{6} = 2n\pi \\ \Rightarrow x &= 2n\pi + \frac{\pi}{6}, n \in \mathbb{I} \text{ fnf.} \end{aligned}$$

CHALLENGER QUESTION

Q. Solve:

$$3\sqrt{3} \sin^3 x + \cos^3 x + 3\sqrt{3} \sin x \cos x = 1$$

$$\begin{aligned} \text{Sol. } (\sqrt{3} \sin x)^3 + (\cos x)^3 + (-1)^3 &= 0 \\ = 3(\sqrt{3} \sin x)(\cos x)(-1) &= 0 \\ \text{If } a^3 + b^3 + c^3 = 3abc & \\ \Rightarrow a = b = c \text{ (or) } a + b + c = 0 & \\ \Rightarrow \sqrt{3} \sin x = \cos x = -1 &\Rightarrow \text{Not possible} \\ \Rightarrow \sqrt{3} \sin x + \cos x - 1 = 0 & \\ \Rightarrow \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x = \frac{1}{2} & \\ \Rightarrow \cos \left(x - \frac{\pi}{3} \right) = \cos \frac{\pi}{3} &\Rightarrow x - \frac{\pi}{3} = 2n\pi \pm \frac{\pi}{3} \\ \Rightarrow x \in \{2n\pi\} \cup \left\{ (3n+1) \frac{2\pi}{3} \right\}, n \in \mathbb{I} \text{ fnf.} \end{aligned}$$

Critical Thinking Question

Q. Find the number of solution of the equation in $[0, 2\pi]$, $\tan(5\pi \cos \alpha) = \cot(5\pi \sin \alpha)$.

$$\begin{aligned} \text{Sol. } \tan \theta_1 &= \cot \theta_2 \Rightarrow \tan \theta_1 = \tan \left(\frac{\pi}{2} - \theta_2 \right) \\ \theta_1 &= n\pi + \frac{\pi}{2} - \theta_2 \\ \theta_1 + \theta_2 &= n\pi + \frac{\pi}{2} \end{aligned}$$

$$5\pi \cos \alpha + 5\pi \sin \alpha = n\pi + \frac{\pi}{2}$$

$$\Rightarrow \cos \alpha + \sin \alpha = \frac{2n+1}{10}$$

$$\Rightarrow -\sqrt{2} \leq \frac{2n+1}{10} \leq \sqrt{2}$$

$$\Rightarrow -10\sqrt{2} \leq 2n+1 \leq 10\sqrt{2}$$

$$\Rightarrow \frac{-10\sqrt{2}-1}{2} \leq n \leq \frac{10\sqrt{2}-1}{2}$$

$$n \in \{-7, -6, -5, \dots, 0, 1, \dots, 6\}$$

if, $n = 0$

$$\Rightarrow \cos \alpha + \sin \alpha = \frac{1}{10} \Rightarrow 2 \text{ solutions}$$

If, $n = -6$

$$\Rightarrow \cos \alpha + \sin \alpha = \frac{-11}{10} \Rightarrow 2 \text{ solutions}$$

Similarly for all 14 values of n

$$\text{fnf} \Rightarrow 14 \times 2 = 28 \text{ sol}^n.$$

Question [JEE Main-2023]

Q. Let $S = \{\theta \in [0, 2\pi) : \tan(\pi \cos \theta) + \tan(\pi \sin \theta) = 0\}$. Then $\sum_{\theta \in S} \sin^2 \left(\theta + \frac{\pi}{4} \right)$ is equal to

Sol. $\tan m + \tan n = 0$

$$\tan m = -\tan n \Rightarrow \tan m = \tan(-n)$$

$$\tan(\pi \cos \theta) = \tan(-\pi \sin \theta)$$

$$\pi \cos \theta = n\pi + (-\pi \sin \theta)$$

$$\sin \theta + \cos \theta = n$$

$$\Rightarrow \sin \left(\theta + \frac{\pi}{4} \right) = \frac{n}{\sqrt{2}}$$

$$\frac{n}{\sqrt{2}} = \frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$$

$$\sin \left(\theta + \frac{\pi}{4} \right) = \frac{-1}{\sqrt{2}} \rightarrow \theta + \frac{\pi}{4} = \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$\sin \left(\theta + \frac{\pi}{4} \right) = 0 \rightarrow \theta + \frac{\pi}{4} = \pi, 2\pi$$

$$\sin \left(\theta + \frac{\pi}{4} \right) = \frac{1}{\sqrt{2}} \rightarrow \theta + \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$\sum_{\theta \in S} \sin^2 \left(\theta + \frac{\pi}{4} \right)$$

Santosh: $\frac{1}{2} + 0 + \frac{1}{2}$ (wrong)

Sumit: $\frac{1}{2} + \frac{1}{2} + 0 + 0 + \frac{1}{2} + \frac{1}{2}$ (Right)

f.n.f. = 2

General doubt

Doubt-01: $\sin \theta = \frac{1}{2}$
 $\cos \theta = \frac{\sqrt{3}}{2}$ } Same ?? $\rightarrow$ NO

Doubt-02: $\sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \dots, \infty$
 $\theta = \sin^{-1} \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

Question [JEE Main-2021]

Q. The number of solution of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is

Sol. Define modulus first

If $\theta \in I$ & III : $\cot x = \cot x + \frac{1}{\sin x}$

$0 = \frac{1}{\sin x} \rightarrow \phi$

If $\theta \in II$ & IV : $-\cot x = \cot x + \frac{1}{\sin x}$

$-2 \cot x = \frac{1}{\sin x}$

$\frac{-2 \cos x}{\sin x} = \frac{1}{\sin x} \Rightarrow \cos x = \frac{-1}{2}$

$\Rightarrow x \rightarrow II \text{ quad.} \Rightarrow x = \frac{2\pi}{3}$

$\Rightarrow$ one solution f.n.f.

Question [JEE (Adv.)-2021]

Q. The positive integer value of $n > 3$ satisfying the equation $\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

Sol. मान लो: $\frac{\pi}{n} = \theta$

$\frac{1}{\sin \theta} = \frac{1}{\sin 2\theta} + \frac{1}{\sin 3\theta}$

$\Rightarrow \frac{1}{\sin \theta} - \frac{1}{\sin 3\theta} = \frac{1}{\sin 2\theta}$

$\Rightarrow \frac{\sin 3\theta - \sin \theta}{\sin \theta \sin 3\theta} = \frac{1}{2 \sin \theta \cos \theta}$

$\Rightarrow 2 \cos 2\theta \sin \theta = \frac{\sin 3\theta}{2 \cos \theta}$

$\Rightarrow 2 \cos 2\theta \sin \theta \cdot 2 \cos \theta = \sin 3\theta$

$\Rightarrow 2 \cos 2\theta \sin 2\theta = \sin 3\theta$

$\Rightarrow \sin 4\theta = \sin 3\theta \Rightarrow 4\theta = m\pi + (-1)^m(3\theta)$

If $m = \text{Odd}$

$4\theta = m\pi - 3\theta$

$7\theta = m\pi \Rightarrow \frac{7\pi}{n} = \pi, 3\pi, 5\pi, 7\pi, \dots$

$\Rightarrow n = 7$ f.n.f.

If $m = \text{even}$

$4\theta = m\pi + 3\theta$

$\theta = m\pi \Rightarrow \frac{\pi}{n} = 0, 2\pi, 4\pi, \dots \Rightarrow n \in \phi$

Type-03

Solving equations by transforming a sum of trigonometric functions into a product.

Question [JEE Main-2021]

Q. The sum of all values of x in $[0, 2\pi]$, for which $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$, is equal to

(A) 11π

(B) 9π

(C) 8π

(D) 12π

Sol. $2 \sin 2x \cos x + 2 \sin 3x \cos x = 0$

$2 \cos x \{\sin 2x + \sin 3x\} = 0$

$2 \cos x \cdot 2 \sin \frac{5x}{2} \cos \frac{x}{2} = 0$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

$$\text{or } \sin \frac{5x}{2} \Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi$$

$$\text{or } \cos \frac{x}{2} = 0 \Rightarrow x = \pi$$

$$\text{Sum} = 2\pi + 4\pi + 2\pi + \pi = 9\pi \text{ fnf.}$$

Question

Q. General solution of $\sin x + \sin 5x = \sin 2x + \sin 4x$ is DIBY!!

Sol. $\left(\frac{n\pi}{3}\right)$

Type-04

Solving equations by transforming a product of trigonometric functions into a sum.

Question

Q. Solve: $\cos x \cdot \cos 2x \cdot \cos 3x = \frac{1}{4}$

Sol. $2(2 \cos x \cdot \cos 2x) \cos 3x = 1$
 $\Rightarrow 2(\cos(3x) + \cos(x)) \cos 3x = 1$
 $\Rightarrow 2 \cos^2 3x - 1 + 2 \cos x \cos 3x = 0$
 $\Rightarrow \cos(6x) + \cos 4x + \cos 2x = 0$
 $\Rightarrow 2 \cos 4x \cos 2x + \cos 4x = 0$
 $\Rightarrow (\cos 4x)\{2 \cos 2x + 1\} = 0$
 $\cos 4x = 0 \cup \cos 2x = -\frac{1}{2} = \cos \frac{2\pi}{3}$
 $4x = (2n+1)\frac{\pi}{2} \cup 2x = \left(2n\pi + \frac{2\pi}{3}\right)$
 $\text{fnf} \Rightarrow x \in \left\{(2n+1)\frac{\pi}{8}\right\} \cup \left\{(3n+1)\frac{\pi}{3}\right\}$

Question

Q. Number of solution of the trigonometric equation in $[0, \pi]$ is $\sin 3\theta = 4 \sin \theta \cdot \sin 2\theta \cdot \sin 4\theta$.

(A) 4 (B) 16 (C) 8 (D) 0

Sol. $3 \sin \theta - 4 \sin^3 \theta = 2 \cdot (2 \sin \theta \sin 4\theta)$
 $(2 \sin \theta \cos \theta)$
 $\Rightarrow \sin \theta(3 - 4 \sin^2 \theta) = 4(\cos 3\theta - \cos 5\theta)$
 $\sin \theta \cos \theta$ and $(\sin \theta = 0)$... (1)
 $\Rightarrow 3 - 4 \sin^2 \theta = 2(2 \cos 3\theta \cos \theta - 2 \cos 5\theta \cos \theta)$
 $\Rightarrow 3 - 4 \sin^2 \theta = 2(\cos 2\theta + \cos 4\theta - (\cos 4\theta + \cos 6\theta))$

$$\Rightarrow (3 - 4 \sin^2 \theta) - 2(\cos 2\theta - \cos 6\theta) = 0$$

$$\Rightarrow 3 - 4 \sin^2 \theta - 2 + 4 \sin^2 \theta + 2 \cos 6\theta = 0$$

$$\Rightarrow 1 + 2 \cos 6\theta = 0 \quad \dots (2)$$

$$\Rightarrow \sin \theta = 0 \Rightarrow \theta = 0, \pi$$

$$\Rightarrow \cos 6\theta = -\frac{1}{2} = \cos \frac{2\pi}{3}$$

$$\Rightarrow 6\theta = 2n\pi \pm \frac{2\pi}{3} \Rightarrow \theta = (3n \pm 1)\frac{\pi}{9}$$

$$\Rightarrow \theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}$$

No. of solutions in $[0, \pi]$ is 8 fnf.

Type-05

Solving equations by substitution

Form $f(\sin x \pm \cos x, \sin x \cdot \cos x) = 0$

Put $\sin x \pm \cos x = t$

Use: $\sin 2x = 1 - (\sin x - \cos x)^2$
 $\sin 2x = (\sin x + \cos x)^2 - 1$

Question

Q. Solve the equation:

$$\sin x + \cos x - 2\sqrt{2} \sin x \cos x = 0.$$

Sol. Sunita: $\sin x + \cos x = t$

$$\text{eqn.} \rightarrow t - \sqrt{2}(t^2 - 1) = 0$$

$$\Rightarrow t - \sqrt{2}t^2 + \sqrt{2} = 0 \Rightarrow \sqrt{2}t^2 - t - \sqrt{2} = 0$$

$$\Rightarrow (t - \sqrt{2})(\sqrt{2}t + 1) = 0 \Rightarrow t = \sqrt{2}, -\frac{1}{\sqrt{2}}$$

$$\sin x + \cos x = \sqrt{2} \rightarrow \text{Type-II}$$

$$\sin x + \cos x = -\frac{1}{\sqrt{2}} \rightarrow \text{Type-II DIBY!!}$$

Sushmita: $\sin x + \cos x = \sqrt{2} \sin 2x$

$$\Rightarrow \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \sin 2x$$

$$\sin\left(x + \frac{\pi}{4}\right) = \sin 2x \quad \text{DIBY!!}$$

Question

Q. Solve: $\sin^2 x + (\sqrt{3} - 1) \sin x \cos x - \sqrt{3} \cos^2 x = 0.$

Sol. Sunita:

$$\begin{aligned} \frac{1}{2} - \frac{\cos 2x}{2} + \left(\frac{\sqrt{3}-1}{2} \right) \sin 2x - \sqrt{3} \left(\frac{1}{2} + \frac{\cos 2x}{2} \right) &= 0 \\ \Rightarrow \frac{1}{2} - \frac{1}{2} \cos 2x + \frac{\sqrt{3}-1}{2} \sin 2x - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cos 2x &= 0 \\ \Rightarrow \left(\frac{\sqrt{3}-1}{2\sqrt{2}} \right) \sin 2x - \left(\frac{\sqrt{3}+1}{2\sqrt{2}} \right) \cos 2x &= \frac{\sqrt{3}-1}{2\sqrt{2}} \\ \Rightarrow (\sin 15^\circ) \sin 2x - (\cos 15^\circ) \cos 2x &= (\sin 15^\circ) \\ \Rightarrow \sin(2x - 15^\circ) &= \sin 15^\circ \quad \text{DIBY!!} \end{aligned}$$

Sushmita: Divide by $\cos^2 x$

$$\begin{aligned} \frac{\sin^2 x}{\cos^2 x} + (\sqrt{3}-1) \frac{\sin x \cos x}{\cos^2 x} - \frac{\sqrt{3} \cos^2 x}{\cos^2 x} &= 0 \\ \Rightarrow \tan^2 x + (\sqrt{3}-1) \tan x - \sqrt{3} &= 0; \\ \text{put: } \tan x = t & \\ \Rightarrow t^2 + \sqrt{3}t - t - \sqrt{3} &= 0 \\ \Rightarrow t(t + \sqrt{3}) - 1(t + \sqrt{3}) &= 0 \\ \Rightarrow (t-1)(t + \sqrt{3}) &= 0 \\ \Rightarrow \tan x = 1 \text{ or } \tan x = -\sqrt{3}. \end{aligned}$$

Question [JEE Main-2022]

Q. The number of element in the set $S = \{\theta \in [-4\pi, 4\pi]: 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2 \theta + 5\} = 0$

Sol. 32 DIBY!!

Question [JEE Main-2022]

Q. The number of solutions of the equation

$$\cos\left(\frac{\pi}{3} + x\right) \cos\left(\frac{\pi}{3} - x\right) = \frac{1}{4} \cos^2 2x,$$

$x \in [-3\pi, 3\pi]$ is

- (A) 8 (B) 5
(C) 6 (D) 7

Sol. (D) DIBY!!

Type-06(a)

Type-06(a): Solving equations with the use of **boundedness** of the function.

Max./Min. value

or By comparing range of both side

Question [JEE Main-2024]

Q. The number of solution of the equation $4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$ is

- (A) 1 (B) 3
(C) 2 (D) 0

Sol. $4(1 - \cos^2 x) - 4\cos^3 x + 9 - 4\cos x = 0$
 $\Rightarrow 4 - 4\cos^2 x - 4\cos^3 x + 9 - 4\cos x = 0$
 $\Rightarrow 13 = 4\cos^2 x + 4\cos^3 x + 4\cos x$
 $\Rightarrow \text{Max. value of RHS} = 12 \text{ when } \cos x = 1$
 $\Rightarrow 13 \neq 12 \Rightarrow \text{No solution.}$

Question [JEE (Adv.)-2014]

Q. For $x \in (0, \pi)$ the equation:

$\sin x + 2\sin 2x - \sin 3x = 3$ has

- (A) Infinitely many solutions
(B) Three solutions
(C) One solution
(D) No solution

Sol. $2\sin 2x - (\sin 3x - \sin x) = 3$
 $\Rightarrow 2(2\sin x \cos x) - 2\cos 2x \cdot \sin x = 3$
 $\Rightarrow 2(\sin x)\{2\cos x - \cos 2x\} = 3$
 $\Rightarrow (\sin x)\{2\cos x - (2\cos^2 x - 1)\} = 3$
 $\Rightarrow (\sin x)\left\{3 - 4\left(\cos x - \frac{1}{2}\right)^2\right\} = 3$
 Only True iff $\Rightarrow \sin x = 1 \cap \cos x = \frac{1}{2} \Rightarrow \phi$

Question [JEE Main-2019]

Q. All the pairs (x, y) that satisfy the inequality

$$\left(2^{\sqrt{\sin^2 x - 2\sin x + 5}} \right) \cdot \frac{1}{4\sin^2 y} \leq 1$$

also satisfy the equation

- (A) $2|\sin x| = 2\sin y$ (B) $2\sin x = \sin y$
(C) $\sin x = 2\sin y$ (D) $\sin x = |\sin y|$

Sol. $2^{\sqrt{(\sin x - 1)^2 + 4}} \leq 4\sin^2 y$

$\Rightarrow \text{Min. value of LHS} = 4$

$\Rightarrow \text{Max. value of RHS} = 4$

$\Rightarrow \text{for equality: } \text{LHS} = \text{RHS} = 4$

$\Rightarrow \sin x = 1 \text{ and } \sin^2 y = 1$

$\Rightarrow \sin y = \pm 1 \Rightarrow \sin x = |\sin y| \text{ fnf.}$

Q. Find value of λ for which equation $\sin^6 x + \cos^6 x = \lambda - 1$ is consistent.

Sol. $LHS = \sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3$
 $= (\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x)$
 $= (1)(1 - 2 \sin^2 x \cos^2 x)$

$$LHS = \left(1 - \frac{3}{4} \sin^2 2x\right) \in \left[\frac{1}{4}, 1\right]$$

For consistent range of RHS should same:

$$\Rightarrow \frac{1}{4} \leq \lambda - 1 \leq 1$$

$$\frac{5}{4} \leq \lambda \leq 2 \text{ fnf.}$$

Question [JEE Main-2023]

Q. The set of all values of λ for which the equation $\cos^2 2x - 2 \sin^4 x - 2 \cos^2 x = \lambda$.

- (A) $[-2, -1]$ (B) $\left[-2, -\frac{3}{2}\right]$
 (C) $\left[-1, -\frac{1}{2}\right]$ (D) $\left[-\frac{3}{2}, -1\right]$

Sol. $LHS = \cos^2 2x - 2 \left(\frac{1 - \cos 2x}{2}\right)^2 - 2 \left(\frac{1 + \cos 2x}{2}\right)^2$

$$LHS = \frac{\cos^2 2x - 3}{2} \in \left[\frac{-3}{2}, -1\right]$$

$\lambda \in \text{Range of LHS.}$

Acidic Question

Q. Solve: $\sin^{23} x + \cos^{77} x = 1$

Sol. $\Rightarrow \sin^{23} x \leq \sin^2 x \dots(i)$

$\Rightarrow \cos^{77} x \leq \cos^2 x \dots(ii)$

add (i) & (ii):

$$\Rightarrow \sin^{23} x + \cos^{77} x \leq 1$$

Equality $\Rightarrow \sin^{23} x = \sin^2 x \cap \cos^{77} x = \cos^2 x$

$$\Rightarrow \sin^{23} x = \sin^2 x \Rightarrow \sin x = 0, 1$$

$$\Rightarrow \cos^{77} x = \cos^2 x \Rightarrow \cos x = 1, 0$$

$$\text{fnf} \Rightarrow x \in \left\{(4n+1)\frac{\pi}{2}\right\} \cup \{2n\pi\} \quad n \in \mathbb{Z}$$

Question [JEE Main-2021]

Q. The number of solution of $\sin^7 x + \cos^7 x = 1$, $x \in [0, 4\pi]$ is equal to

Sol. 5 DIBY!!

Type-06(b)

When different types of variables or functions are present.

Black बाबल (KKPNCC)

Same type or variety of functions को ek side रखें, now compare range of LHS & RHS.

Question

Q. Solve for x and y :

(i) $12 \sin x - 2y^2 = 21 - 8y - 5 \cos x$

(ii) $x^2 + 2x \sin(xy) + 1 = 0$

Sol. (i) $12 \sin x + 5 \cos x = (2y^2 - 8y + 21)$

$$\Rightarrow RHS = 2(y^2 - 4y + 4) + 13$$

$$\Rightarrow RHS = 2(y - 2)^2 + 13$$

$$\Rightarrow RHS \in [13, \infty) \text{ \& } LHS \in [-13, 13]$$

$$\Rightarrow \text{For equality: } LHS = RHS = 13$$

$$\Rightarrow RHS = 13 \Rightarrow y = 2$$

$$\Rightarrow LHS = 12 \sin x + 5 \cos x = 13$$

$$\Rightarrow \frac{12}{13} \sin x + \frac{5}{13} \cos x = 1$$

$$\Rightarrow \cos(x - \alpha) = 1$$

$$\Rightarrow x = 2n\pi + \alpha; \alpha = \cos^{-1} \frac{5}{13}$$

$$(x = 2n\pi + \alpha) \text{ \& } (y = 2) \text{ fnf}$$

(ii) $x^2 + 1 = -2x \sin(xy)$

$$\Rightarrow x + \frac{1}{x} = -2 \sin(xy)$$

$$LHS \in (-\infty, -2] \cup [2, \infty) \text{ \& } RHS \in [-2, 2]$$

$$\text{fnf} \Rightarrow x = \pm 1, y = (4n+3)\frac{\pi}{2}$$

Question [JEE Main-2022]

Q. The number of elements in the set

$$S = \left\{x \in \mathbb{R} : 2 \cos \left(\frac{x^2 + x}{6}\right) = 4^x + 4^{-x}\right\}$$

(A) 1

(B) 3

(C) 0

(D) Infinite

Sol. $\cos\left(\frac{x^2 + x}{6}\right) = \frac{1}{2}\left(4^x + \frac{1}{4^x}\right)$

$\Rightarrow$ Max. value of LHS = 1

$\Rightarrow$ Min. value of RHS = 1

Equality hold when: $\boxed{\text{LHS} = \text{RHS} = 1}$

Equality hold for $x = 0$; so 1 fnf.

Question [JEE Main-2024]

Q. The sum of the solutions $x \in \mathbb{R}$ of the

equation $\frac{3 \cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$

Sol. LHS = $\frac{(\cos 2x)\{3 + \cos^2 2x\}}{(\cos 2x)(\cos^4 x + \sin^4 x + \cos^2 x \sin^2 x)}$

LHS = $\frac{3 + \cos^2 2x}{\left(\frac{3 + \cos^2 2x}{4}\right)} = 4$

So, RHS = 4

$\Rightarrow 4 = x^3 - x^2 + 6 \Rightarrow x^3 - x^2 + 2 = 0$

$\Rightarrow (x+1)(x^2 - 2x + 2) = 0 \Rightarrow \boxed{x = -1}$ fnf.

SOLVING GRAPHICALLY

For graphically solving or finding number of solutions of equation: $\boxed{f(x) = g(x)}$

Step-01: Simplify both side (whose graphs are known or easy to draw)

Step-02: Draw $y = f(x)$ and $y = g(x)$ on same axis.

Step-03: Number of solutions = Number of points of intersection of graph of $f(x)$ and $g(x)$.

Solutions of equation are 'x-coordinates of points of intersection.'

Black बाबल (KKPNCC)

ये Method पूरी Maths में काम आता है

Question [JEE Main-2021]

Q. The number of solutions of equation $x + 2$

$\tan x = \pi/2$ in the interval $[0, 2\pi]$ is

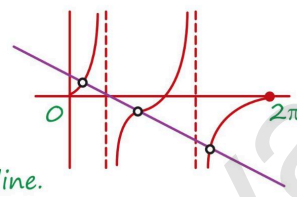
(A) 3 (B) 4 (C) 2 (D) 5

Sol. $2 \tan x = \frac{\pi}{2} - x$

$\tan x = \frac{\pi}{4} - \frac{x}{2}$

$y = \frac{\pi}{4} - \frac{x}{2} \rightarrow$ St line.

$\Rightarrow$ P.O.I. of LHS & RHS = 3 fnf.



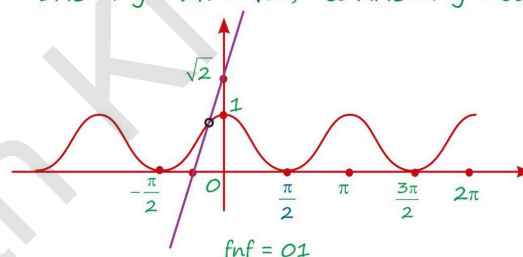
Question [JEE Main-2022]

Q. The number of solutions of the equation $2\theta - \cos^2 \theta + \sqrt{2} = 0$ in $\mathbb{R}$ is equal to ____.

Sol. $2\theta + \sqrt{2} = \cos^2 \theta$

$2x + \sqrt{2} = \cos^2 x$

LHS $\Rightarrow y = 2x + \sqrt{2}$; & RHS $\Rightarrow y = \cos^2 x$



BUMPER CHALLENGER QUESTION

Q. Find number of solution of following equations:

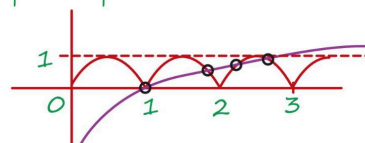
(i) $|\sin \pi x| = \ln x$

(ii) $\min\{\tan x, \cos x\} = \frac{1}{8}, x \in (-2\pi, 2\pi)$

(iii) $2 \sin |x| - |x| + 1 = 0$

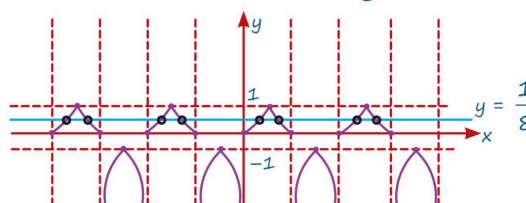
(iv) $3 |\cos x| = 5 |\sin x|, x \in [0, 3\pi]$

Sol. (i) $|\sin \pi x| = \ln x$



Number of solutions = 04 fnf.

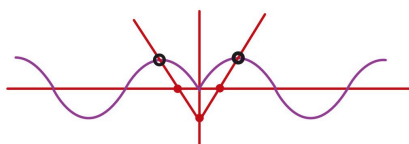
(ii) $\min\{\tan x, \cos x\} = \frac{1}{8}, x \in (-2\pi, 2\pi)$



Number of solutions = 08 fnf.

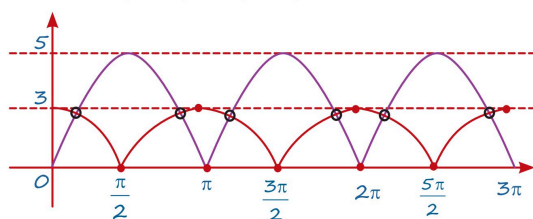
$$(iii) 2 \sin |x| - |x| + 1 = 0$$

$$\sin |x| = \frac{|x| - 1}{2} \Rightarrow y = \frac{|x| - 1}{2}$$



Number of solutions = 02 fnf.

$$(iv) 3|\cos x| = 5|\sin x|, x \in [0, 3\pi]$$



Number of solutions = 06 fnf.

GRAPHICALLY SOLVING INEQUALITY

Inequality: $f(x) > g(x)$

Solution: Draw both graph on same axis. Now answer portion of x-axis for which graph of $f(x)$ lies above graph of $g(x)$.

TRIGONOMETRIC INEQUALITY

- Solve given inequality for 1 complete cycle and generalise the solution
- To generalise: add integral multiple of period.
- $\sin, \cos, \sec, \csc \rightarrow$ add ' $2n\pi$ '
- $\tan, \cot \rightarrow$ add ' $n\pi$ '

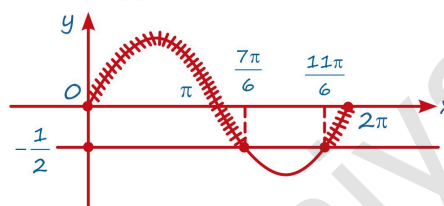
Question

Q. Solving following inequalities:

$$(i) \sin x \geq -\frac{1}{2}$$

$$(ii) |\tan x| > 1$$

$$\text{Sol. (i) } \sin x \geq -\frac{1}{2}$$

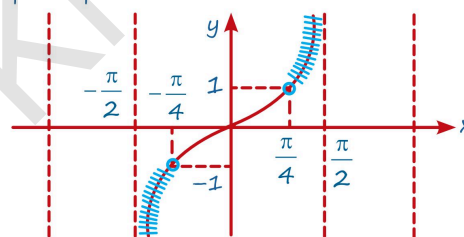


$$\Rightarrow x \in \left[0, \frac{7\pi}{6}\right] \cup \left[\frac{11\pi}{6}, 2\pi\right]$$

Now generalise: add ' $2n\pi$ ', $n \in \mathbb{I}$

$$x \in \left[2n\pi, 2n\pi + \frac{7\pi}{6}\right] \cup \left[2n\pi + \frac{11\pi}{6}, 2n\pi + 2\pi\right]$$

$$(ii) |\tan x| < 1 \Rightarrow \tan x > -1 \cup \tan x < 1$$



$$x \in \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

Now generalise: add ' $n\pi$ ', $n \in \mathbb{I}$

$$x \in \left(n\pi - \frac{\pi}{2}, n\pi - \frac{\pi}{4}\right) \cup \left(n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{2}\right),$$

Question [IIT-JEE-2006]

Q. The set of values of θ satisfying in the equation $2 \sin^2 \theta - 5 \sin \theta + 2 > 0$, where $0 < \theta < 2\pi$, is

$$(A) \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$$

$$(B) \left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, 2\pi\right]$$

$$(C) \left[0, \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}, 2\pi\right]$$

(D) None of the above

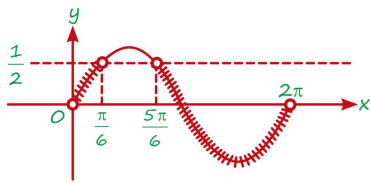
Sol. $\sin \theta = t$

$$2t^2 - 5t + 2 > 0 \Rightarrow (2t - 1)(t - 2) > 0$$

$$(2 \sin \theta - 1)(\sin \theta - 2) > 0$$

$$2 \sin \theta - 1 < 0; (\sin \theta - 2 = -ve \text{ always})$$

$$\Rightarrow \sin \theta < \frac{1}{2}$$



$$\Rightarrow \theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right) \text{ fnf.}$$

Question [JEE (Adv.)-2020]

Q. Let $f: [0, 2] \rightarrow \mathbb{R}$ be the function defined by

$$f(x) = (3 - \sin(2\pi x)) \sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right)$$

If $\alpha, \beta \in [0, 2]$ are such that: $\{x \in [0, 2];$

$f(x) \geq 0\} = [\alpha, \beta]$, then the value of $\beta - \alpha$ is

$$\text{Sol. } f(x) = (3 - \sin 2\theta) \sin\left(\theta - \frac{\pi}{4}\right) - \sin\left(3\theta + \frac{\pi}{4}\right)$$

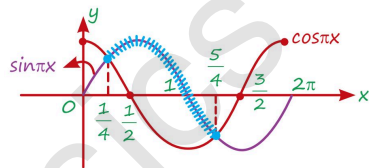
$$\Rightarrow f(x) = (3 - \sin 2\theta) \left(\frac{\sin \theta - \cos \theta}{\sqrt{2}} \right) - \left(\frac{\sin 3\theta + \cos 3\theta}{\sqrt{2}} \right)$$

$$\Rightarrow f(x) = \left(\frac{\sin \theta - \cos \theta}{\sqrt{2}} \right) (\sin 2\theta + 4)$$

$$\Rightarrow f(x) \geq 0 \Rightarrow \left(\frac{\sin \theta - \cos \theta}{\sqrt{2}} \right) (\sin 2\theta + 4) \geq 0$$

$$\Rightarrow \sin \theta \geq \cos \theta$$

$$\Rightarrow \boxed{\sin \pi x \geq \cos \pi x}, \quad x \in [0, 2]$$



$$x \in \left[\frac{1}{4}, \frac{5}{4}\right], \quad \beta - \alpha = \frac{5-1}{4} = 1. \text{ fnf}$$

Miscellaneous Questions

Question [JEE (Adv.)-2015]

Q. The number of distinct solutions of the

$$\text{equation } \frac{5}{4} \cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x +$$

$\sin^6 x = 2$ in the interval $[0, 2\pi]$ is

$$\text{Sol. } \frac{5}{4} \cos^2 2x + (1 - 2 \sin^2 x \cos^2 x)$$

$$+ (1 - 3 \sin^2 x \cos^2 x) = 2$$

$$\Rightarrow \frac{5}{4} \cos^2 2x - \frac{5}{4} \sin^2 2x = 0$$

$$\Rightarrow \frac{5}{4} (\cos^2 2x - \sin^2 2x) = 0$$

$$\Rightarrow \cos 4x = 0 \Rightarrow x = (2n+1) \frac{\pi}{8}$$

$$\Rightarrow x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \dots, \frac{15\pi}{8} \Rightarrow 8 \text{ fnf.}$$

Question [JEE (Adv.)-2016]

Q. Let $S = \left\{ x \in (-\pi, \pi) : x \neq 0, \pm \frac{\pi}{2} \right\}$. The sum

of all distinct solutions of the equation $\sqrt{3} \sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to

(A) $-\frac{7\pi}{9}$

(B) $-\frac{2\pi}{9}$

(C) 0

(D) $\frac{5\pi}{9}$

$$\text{Sol. } \left(\sqrt{3} \frac{1}{\cos x} + \frac{1}{\sin x} \right) + 2 \left(\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} \right) = 0$$

$$\Rightarrow \left(\frac{\sqrt{3} \sin x + \cos x}{\sin x \cos x} \right) + 2 \left(\frac{\sin^2 x - \cos^2 x}{\sin x \cos x} \right) = 0$$

$$\Rightarrow (\sqrt{3} \sin x + \cos x) = 2 \cos 2x$$

$$\Rightarrow \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x = \cos 2x$$

$$\Rightarrow \cos \left(x - \frac{\pi}{3} \right) = \cos 2x$$

$$\Rightarrow 2x = 2n\pi \pm \left(x - \frac{\pi}{3} \right)$$

$$\Rightarrow 2x = 2n\pi + x - \frac{\pi}{3} \Rightarrow \boxed{x = 2n\pi - \frac{\pi}{3}}$$

$$(\text{or}) 2x = 2n\pi - x + \frac{\pi}{3} \Rightarrow \boxed{3x = 2n\pi + \frac{\pi}{3}}$$

$$\text{Sum} = -\frac{\pi}{3} + \frac{\pi}{9} + \frac{7\pi}{9} + \left(\frac{-5\pi}{9} \right) = 0 \text{ fnf.}$$

Critical Thinking Question [JEE Main-2019]

Q. If $\sin^4 \alpha + 4 \cos^4 \beta + 2 = 4\sqrt{2} \sin \alpha \cos \beta$ and $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to

- (A) $\sqrt{2}$ (B) $-\sqrt{2}$
(C) -1 (D) 0

Sol. $AM \geq GM$

$$\Rightarrow (\sin^4 \alpha, 4 \cos^4 \beta, 1, 1)$$

$$\frac{\sin^4 \alpha + 4 \cos^4 \beta + 1 + 1}{4} \geq (\sin^4 \alpha \cdot 4 \cos^4 \beta \cdot 1 \cdot 1)^{1/4}$$

$$\Rightarrow \sin^4 \alpha + 4 \cos^4 \beta + 2 \geq 4\sqrt{2} \sin \alpha \cos \beta$$

$\Rightarrow$ equality hold when:

$$\Rightarrow \sin^4 \alpha = 4 \cos^4 \beta = 1$$

$$\Rightarrow \sin \alpha = \pm 1 \text{ \& } \cos \beta = \pm \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos(\alpha + \beta) - \cos(\alpha - \beta) = -2 \sin \alpha \sin \beta$$

$$\Rightarrow -2(1) \left(\frac{1}{\sqrt{2}} \right) = -\sqrt{2} \text{ fnf.}$$

CHALLENGER QUESTION

Q. Solve: $2 \sin \left(3x + \frac{\pi}{4} \right) = \sqrt{1 + 8 \sin 2x \cdot \cos^2 2x}$

Sol. LHS = $2 \left(\sin 3x \times \frac{1}{\sqrt{2}} + \cos 3x \times \frac{1}{\sqrt{2}} \right)$

$$RHS = \sqrt{1 + 4 \cos 2x (2 \sin 2x \cos 2x)}$$

$$\Rightarrow \sqrt{2} (\sin 3x + \cos 3x)$$

$$= \sqrt{1 + 2(2 \cos 2x \cdot \sin 4x)}$$

$$SBS: \Rightarrow 2(\sin^2 3x + \cos^2 3x + \sin 6x)$$

$$= 1 + 2(\sin 6x + \sin 2x)$$

$$\Rightarrow 2 + 2 \sin 6x = 1 + 2 \sin 6x + 2 \sin 2x$$

$$\Rightarrow \sin 2x = \frac{1}{2}$$

$$\Rightarrow 2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \dots$$

$$\Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \dots$$

$$x \in \left\{ n\pi + \frac{\pi}{12} \right\} \cup \left\{ n\pi + \frac{5\pi}{12} \right\}, \eta \in \mathbb{I}$$

Question [JEE Main-2025]

Q. The sum of all values of $\theta \in [0, 2\pi]$ satisfying $2\sin^2 \theta = \cos 2\theta$ and $2\cos^2 \theta = 3\sin \theta$ is

- (A) π (B) $\frac{5\pi}{6}$
(C) $\frac{\pi}{2}$ (D) 4π

Sol. $2\sin^2 \theta = \cos 2\theta$

$$2\sin^2 \theta = 1 - 2\sin^2 \theta$$

$$4\sin^2 \theta = 1$$

$$\sin^2 \theta = \frac{1}{4}$$

$$\sin \theta = \pm \frac{1}{2}$$

$$2\cos^2 \theta = 3\sin \theta$$

$$2 - 2\sin^2 \theta - 3\sin \theta = 0$$

$$\sin \theta = \frac{1}{2}$$

So common equation which satisfy both

equation is $\sin \theta = \frac{1}{2}$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6} (\theta \in [0, 2\pi])$$

$$\text{sum} = \pi$$

Question [JEE Main-2025]

Q. If $\theta \in \left[-\frac{7\pi}{6}, \frac{4\pi}{3} \right]$, then the number of solutions of $\sqrt{3} \operatorname{cosec}^2 \theta - 2(\sqrt{3} - 1) \operatorname{cosec} \theta - 4 = 0$ is equal to

- (A) 7 (B) 10
(C) 6 (D) 8

Sol. $\sqrt{3} \operatorname{cosec}^2 \theta - 2(\sqrt{3} - 1) \operatorname{cosec} \theta - 4 = 0$,

Using the quadratic formula:

$$\operatorname{cosec} \theta = \frac{2\sqrt{3} - 2 \pm \sqrt{(2(\sqrt{3} - 1))^2 + 4\sqrt{3} \cdot 4}}{2\sqrt{3}}$$

Simplifying inside the square root:

$$(2\sqrt{3} - 1)^2 + 4\sqrt{3} \cdot 4 = 4(3 + 1 - 2\sqrt{3}) + 16\sqrt{3}$$

This simplifies to:

$$4(4 - 2\sqrt{3}) + 16\sqrt{3} = 16 - 8\sqrt{3} + 16\sqrt{3} = 16 + 8\sqrt{3}$$

Therefore, the quadratic gives:

$$\operatorname{cosec} \theta = \frac{2\sqrt{3} - 2 \pm 2(\sqrt{3} + 1)}{2\sqrt{3}}$$

By solving, we find: $\operatorname{cosec} \theta = \frac{-2}{\sqrt{3}}, 2$

Thus, for each value of the cosecant, θ can taken several values that fall within the given interval $\left[-\frac{7\pi}{6}, \frac{4\pi}{3}\right]$:

$$\theta = \frac{-7\pi}{6}, \frac{-2\pi}{6}, \frac{-\pi}{3}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{4\pi}{3}$$

$\Rightarrow 6$ (solutions) fnf.

Question [JEE Main-2016]

Q. If $\theta \in [-2\pi, 2\pi]$, then the number of solutions of $2\sqrt{2} \cos^2 \theta + (2 - \sqrt{6}) \cos \theta - \sqrt{3} = 0$, is equal to

- (A) 8 (B) 6 (C) 10 (D) 12

Sol. $2\sqrt{2} \cos^2 \theta + (2 - \sqrt{6}) \cos \theta - \sqrt{3} = 0$

$$2\sqrt{2} \cos^2 \theta + 2 \cos \theta - \sqrt{6} \cos \theta - \sqrt{3} = 0$$

$$(2 \cos \theta - \sqrt{3})(\sqrt{2} \cos \theta + 1) = 0$$

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \text{ or } \cos \theta = \frac{-1}{\sqrt{2}}$$

$$\theta = \left\{ \frac{-11\pi}{6}, \frac{-5\pi}{4}, \frac{-3\pi}{4}, \frac{-\pi}{6}, \frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{6} \right\}$$

$\Rightarrow 8$ (solution) fnf.

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